THE ECCENTRIC SLOTTED BALUN

A. NEWMAN, JR.

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by

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PREFACE

Material for this paper was gathered at the Hewlett Packard Co., Palo Alto, California during the winter term of the third year of the U.S. Naval Postgraduate School Engineering Electronics course.

I wish at this time to express my thanks to Mr. Hewlett and Mr. Packard and to the whole Company for their wholehearted cooperation and willing acceptance of me into their organization.

In particular, I would like to express my thanks to Dr. B. M. Oliver, Director of Research, W. B. Wholey, Senior Engineer, and F. E. Barnett, Engineer, for their indulgence and helpful suggestions throughout my stay.

A. Newman, Jr.

Monterey, California
May 22, 1953

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LIST of ILLUSTRATIONS

TABLE of SYMBOLS and ABBREVIATIONS with PAGE FIRST USED

A	position of one slot in w-or z-plane	12
A'	position of second slot in w-or z-plane	12
a	radius of inner conductor in z-plane	12
$a'i$	portion of total axial current in one half of outer conductor	6
a'^2	inverse of transformation ratio	9
α	radius of inner conductor in w-plane	12
b	inner radius of outer conductor in z-plane	12
β	inner radius of outer conductor in w-plane	12
c	absolute value of w - $\lvert w \rvert$	44
C_1	Capacity in one sector from one half of outer to center conductors	13
C_2	Capacity in other sector from one half of outer center conductors	13
C_t	Total capacity in transmission line cross section	13
C	Capacity per unit length in transmission line	10
γ	distance along x-axis from x_2 to the vertical component of a point P in the z-plane	12
d	defined as equal to $\epsilon \rho$, a design parameter	13
Δ	small change	18
δ	distance between conductor centers in z-plane	12
ϵ	eccentricity	14
ϵb	displacement of center conductors	23
θ	angle subtended by slot in z-plane with center at x_2	18
θ'	angle subtended by slot in w-plane	18
i	axial current in transmission line	6
i_b	balanced current in each half of outer conductor	8

H.P.	Hewlett-Packard	30
ln	natural logarithm	14
P	general point in z-plane	12
p	arbitrary point in z-plane along x-axis	11
P_{in}	power incident	9
P_L	power dissipated in load	9
Q	charge	10
Γ	reflection coefficient	8
γ	ratio of b to a	13
φ'	ratio of β to α	18
ϕ	slot width in z-plane	12
ϕ'	slot width in w-plane	18
TEM	transverse electro magnetic	6
VSWR	voltage standing wave ratio	28
V	voltage	10
w	function designating plane	11
x_1	center of inner conductor along x-axis	12
x_2	center of outer conductor along x-axis	12
z	function designating plane	11
Z_L	Balanced load including stub	8
Z_L'	load impedance seen by coaxial section	8
Z_R	unknown load	9
Z_O	characteristic impedance	9

CHAPTER I

INTRODUCTION

1. Summary

The eccentric slotted balun is an eccentric transmission line balance to unbalance transformer with two slots in the outer conductor. The impedance transformation ratio is determined by the degree of eccentricity.

In this paper the theory of the balun is derived and design curves for a fifty ohm unbalanced system to a balanced system with any desired impedance transformation from two to twenty are developed.

The balun was developed at the Hewlett Packard Company for use with their new UHF signal generator for the purpose of aligning UHF television receivers, but by a simple tuning procedure it soon developed into a satisfactory laboratory instrument for other uses such as an impedance measuring device when used with the slotted line. This is the principle the Company will use for their production model.

2. The Problem

There are many instances when it becomes necessary to use an isolation transformer, commonly called a balun, to transform from a balanced system to an unbalanced system. This becomes most evident in the ultra high frequency region where antennas and receiver input terminals are commonly balanced while impedance measuring devices and signal generators have unbalanced terminals.

At lower frequencies this problem is solved quickly by using a wire wound transformer with the proper turns ratio, but as frequency is increased, capacitive coupling currents between primary and secondary windings become

troublesome. This can be eliminated by using electrostatic shielding between primary and secondary. As frequency is further increased, it is found that the shield is no longer effective. The ground plane has become a circuit component because its physical size is no longer small compared to a wavelength. Lead inductance and wiring capacities now have to be accounted for.

When the no-man's land called the UHF region is reached, it is found that normal lumped constant circuits cease to be practical because of their physical size and difficulty in wiring. Wave guides are impractical because wavelengths are too long. For the most part very small and unreliable lumped constants or transmission lines must be used.

Many transmission line transformers are in use, but when used as baluns, the impedance transformation ratio is generally restricted to one to one, or four to one. In the UHF television band the primary interest is transforming a fifty ohm generator unbalanced output terminal to a three hundred ohm balanced receiver input terminal for alignment purposes, or the inverse process to measure balanced antenna impedances on a bridge or slotted line with unbalanced terminals. This obviously requires a six to one impedance transformation.

3. The Slotted Balun

The slotted balun developed by RCA[13] will transform balanced impedances to unbalanced with a transformation ratio of four to one.

The slotted balun is a coaxial line with two diametrically opposing slots milled in the outer conductor. The slot is one quarter wavelength long at the center frequency. Provisions can be made to tune the balun by varying the length of the slots with a shorting strip. At the balanced end

of the balun one half of the outer conductor is shorted to the center conductor. The balanced load is then placed between the two outer conductors.

It has been shown (13) that an unbalanced source will produce balanced currents in a balanced load over the frequency range of 500 to 900 mcs. Theoretically, this balance condition is independent of frequency, but the transformed impedance will be that of the load alone only when the slot length is one quarter wavelength.

It will be shown that the transformation ratio is dependent upon the ratio of currents in the outer conductors. Because of the symmetry of the system, current division can be effected by placing the slots assymetrically on the outer conductor, and the ratio of currents will be the ratio of the angles subtended by the segments of the outer conductors.

4. The Eccentric Slotted Balun

The same effect can be obtained by displacing the outer conductor of the balun and keeping the slots diametrical. The inner diameter of the outer conductor must also be increased slightly to keep the characteristic impedance of the eccentric line the same as the rest of the coaxial system. Proof of this is given in Chapter II.

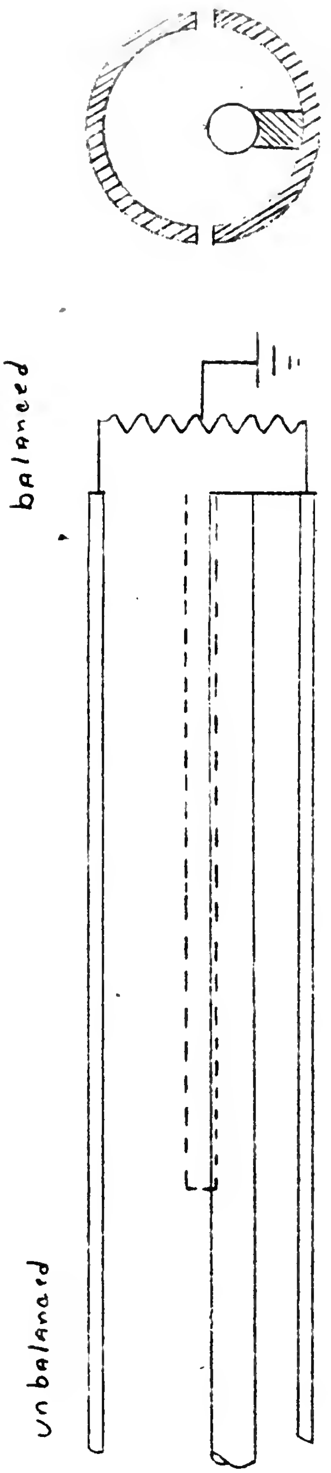
It is desirable to divide the outer conductor currents in this manner because the two outer conductor halves are part of the balanced system, and geometrical symmetry insures balance conditions independent of frequency.

By means of conformal transformation, it is possible to transform the eccentric case to the coaxial case and analysis is then the same as in the coaxial case. Theoretical design curves were constructed, and two baluns were designed from them. The first was a concentric slotted balun

with a four to one impedance transformation, and the second was an eccentric slotted balun with a six to one impedance transformation.

A reliable balanced load was not available, and time did not permit design of one, so the same load was used for both baluns.

The best results, without assumptions, were obtained when the input impedances in both cases were compared at the same frequency. The ratio of these impedances should be in the ratio of six to four. Experimental results show that this ratio varied by 3%.



Eccentric Slotted Balun

Figure 1

CHAPTER II

THEORY

1. The Concentric Slotted Balun

In the slotted section of a concentric slotted balun two TEM modes may be propagated simultaneously. One is the normal coaxial, or unbalanced, mode, and the other is a balanced mode with currents equal in magnitude but opposite in phase in the outer conductors.

The slot merely increases the characteristic impedance slightly for the unbalanced mode. If the slot is narrow, the line can be considered well shielded.[7]

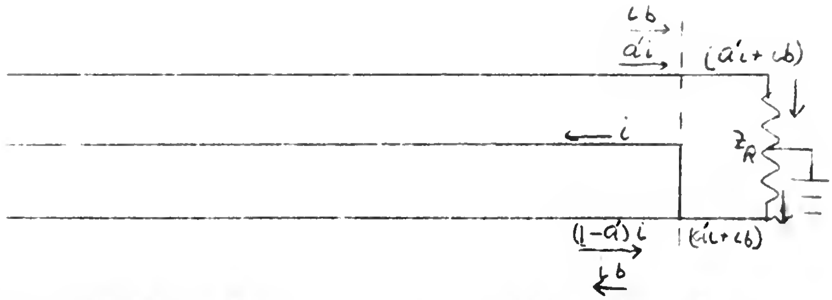
For the balanced mode, the slot provides a balanced two wire line consisting of the two outer conductor halves. This mode is totally reflected at the unbalanced end of the slot because the balanced line is short circuited at the point where the slot terminates and the system becomes coaxial.

For the purpose of this discussion, the generator is assumed to be at the unbalanced end of the balun, and the load is at the balanced end.

Kirchoff's law in a transmission line must hold in a plane as well as at a point. Thus, if there is a current of i in the center conductor, a current of $a'i$ in one of the outer conductors opposite in phase to i, then a current of $(1-a')i$ must flow in the other outer conductor in phase with . See figure 2. These are the normal unbalanced currents.

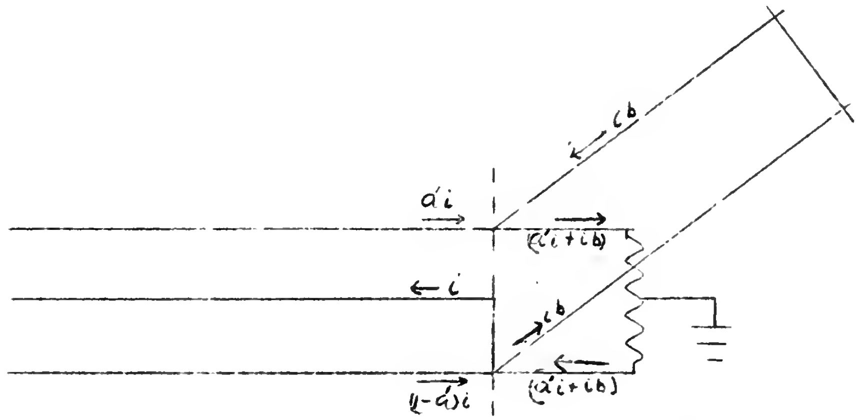
If the above currents are considered as existing at the load end of the slot along with the balanced currents i_b, it is seen that the resultant currents in a balanced load at this point are opposite in phase and equal to

$$(a'i + i_b)$$



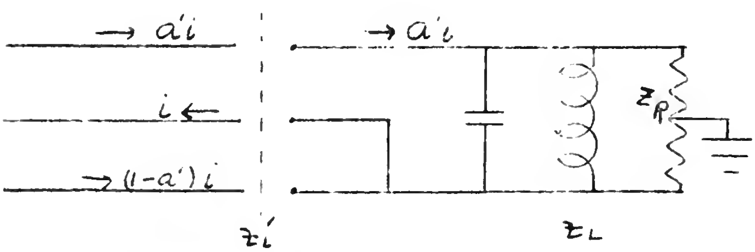
Currents at load junction of slotted balun.

Figure 2



Currents at load junction of slotted balun with balanced line considered as a stub.

Figure 3



Load seen by coaxial section of balun.

Figure 4

which is the condition of balance.

If the current equations for a coaxial line with a shorted stub at the load terminals are now set down (figure 3), it is seen that the load currents are identical with the above case. Thus, the balun can be considered as a slotted coaxial line with a load consisting of the balanced load in parallel with a shorted balanced line. The equivalent circuit for the load will then be a tank circuit in parallel with the balanced load. See figure 4. This total load will be called Z_L.

The short circuit at the load end of the slot can be considered as part of the load circuit. Its purpose is to apply the vector sum of the center conductor and one of the outer conductor currents to the load. The balanced load currents are obtained in this manner.

By considering the balanced mode as acting in the stub alone, the balanced and unbalanced modes can be considered independently, and the slotted section can be analyzed as if it were a coaxial line propagating in the normal TEM coaxial mode with a load Z_L. ($Z_L' \neq Z_L$ because of the impedance transformation).

If a transmission line is not terminated in its characteristic impedance, reflections will take place and standing waves will result, so all of the incident power is not absorbed in the load.

If the reflection coefficient Γ is defined as

$$\Gamma = \frac{Z_L' - Z_o}{Z_L' + Z_o}$$

where Z_L is the apparent load seen by the coaxial line, and Z_o is the characteristic impedance of the coaxial line.

From transmission line theory

$$Z_L' = Z_0 \frac{(1+\Gamma)}{(1-\Gamma)}$$

If the total complex power delivered to the load is considered, it can be seen from figure 4 that

$$P_{in} = i^2 Z_L'$$

and the power absorbed by the load is

$$P_L = a'^2 i^2 Z_L$$

These two powers must be equal, so

$$Z_L = \frac{Z_L'}{a'^2}$$

So that when the balun is used as an impedance measuring device, the load (Z_L') seen by the coaxial line is determined by ordinary slotted line techniques and the unknown Z_L is found by multiplying by $\frac{1}{a'^2}$.

To find the unknown load Z_R, the shorted balanced line must have an infinite input impedance. This will occur when the effective slot length is one quarter wavelength. This means that the balun will have to be tuned.

Tuning can be accomplished with a sliding shorting band around the outer conductor. Calibration is accomplished by slotted line techniques with a known resistive load. The short is moved to a position such that the voltage minima for both a short circuit load and the known resistive load occur at the same point in the slotted line. This position of the short corresponds to a slot length of one quarter wavelength for that particular frequency. This can be repeated at other frequencies, and the outer conductor

can be marked so the tuning procedure is simple. The balun can then be used to measure or transform any impedance.

To achieve a desired impedance transformation, the amount of current in each section of the outer conductor must be controlled.

For a coaxial line propagating in the coaxial TEM mode we know that

$$\frac{\partial I}{\partial z} = - C \frac{\partial V}{\partial t}$$

or axial current is proportional to capacitance[9].

It has been shown[9] that the analysis of a transmission line is valid when made on the basis of static inductance and capacitance, so if the ratio of current in a segment of the outer conductor to the current in the center conductor is considered it is seen that this ratio will be the same as the ratio of capacity in the segment to the total capacity.

The capacitance for two concentric cylindrical conductors is derived by Jordan[3] on a uniform charge basis. From

$$C = \frac{Q}{V}$$

and assuming a uniform charge distribution, it is readily seen that the ratio of capacity in one segment to the total capacity will be the ratio of charge contained in that segment to the total charge. This is the angle subtended by the segment divided by the total angle 2π.

This shows that the current in the two segments may be varied by placing the slots assymetrically around the outer conductor such that the subtended angle between slots is a' times 2π. For an impedance transformation greater than four, the segment with the smaller subtended angle is shorted to the center conductor, and for impedance transformations less than four,

the larger segment is shorted.

When the slots are placed assymetrically around the outer conductor, geometrical symmetry of the balanced system is destroyed.

2. The Eccentric Slotted Balun

In order to maintain the desired symmetry in the balanced system, the eccentric slotted balun was devised. By displacing the outer conductor and maintaining the slots diametrically opposite, a displacement can be chosen which will give the proper current division in the outer conductor halves, and thus the desired impedance transformation ratio. This is again done by choosing the ratio of capacity between the center conductor and one half of the outer conductor to the total capacity, but the charge distribution is no longer uniform, so other means must be used to find the ratio of capacities.

If conformal transformations are now used, and it is recognized that during the process of conformal transformation, field and flux relations are not changed, it is seen that the transformation

$$w = \frac{z - p}{z + p}$$

will transform eccentric circles in the z-plane to concentric circles in the w-plane. A semi-circle in the z-plane will then transform into an arc in the w-plane, the size of the arc is dependent upon the degree of eccentricity. Proof of this is in Appendix I. See figures 5 and 6.

The eccentric case thus transforms to the coaxial case in the w-plane, so it is necessary only to proceed as above and then transform back to the z-plane for the final result. It has been shown that current division in the coaxial case can be effected by placing the slots assymetrically around the outer conductor. If the slots are diametrically opposite in the z-plane,

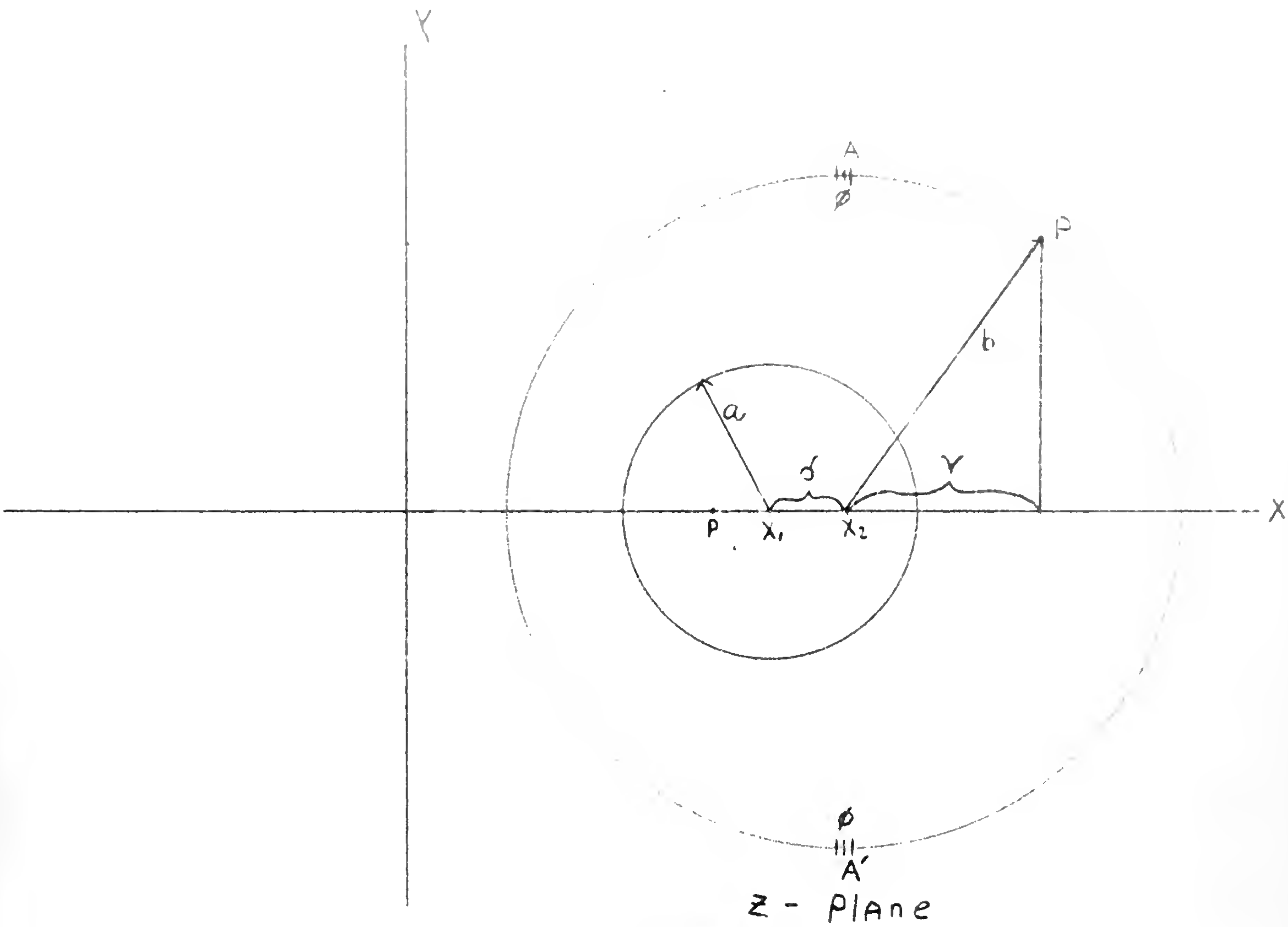


Figure 5

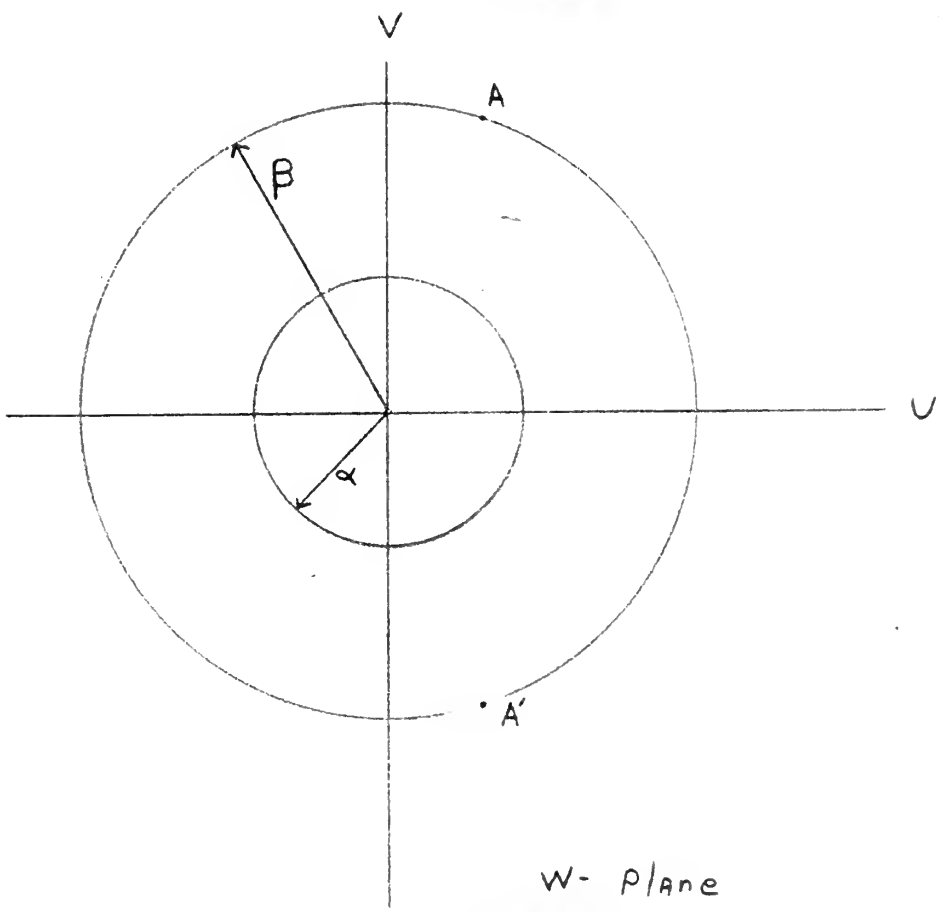


Figure 6

they will be transformed assymetrically to the w-plane. To determine the current division in the outer conductors, it is sufficient to find the angle subtended by the segment of the outer conductor in the w-plane, because in the coaxial case charge distribution is assumed uniform.

Take the point A in the z-plane corresponding to the position of one slot of width $\emptyset$ (figure 5), then

$$z = x_2 + jb$$

This is transformed to a point in the w-plane

$$w = u + jv$$

From Appendix I it is seen that this corresponds to an angle

$$\mathrm{ARG}\, w = \tan^{-1} \frac{1}{2\rho d} \sqrt{(\rho^2 - 1 - d^2)^2 - 4d^2}$$

This is a symmetrical transformation, so that the other slot at A' will be transformed to a point

$$w = u - jv$$

The total angle subtended will be 2ARG w.

If C_1 is the capacitance in the smaller arc, C_2 is the capacitance in the larger arc, and C_t is the total capacitance, then the ratio of segment capacities to total capacity will be the ratio of angles subtended to 2π, or

$$\frac{C_1}{C_t} = \frac{1}{\pi} \cdot \tan^{-1} \frac{1}{2\rho d} \sqrt{(\rho^2 - 1 - d^2)^2 - 4d^2}$$

It follows that

$$\frac{C_2}{C_t} = \left(1 - \frac{C_1}{C_t}\right)$$

3. Characteristic Impedance of the Eccentric Line

The characteristic impedance of a coaxial line is well known, but in this case it is coaxial only in the w-plane.

$$Z_o = 60 \ln \frac{\beta}{\alpha}$$

Transforming to the z-plane, see Appendix II.

$$Z_o = 60 \ln \left[\rho - \frac{1}{2\rho}(\rho^2 - 1 + d^2) + \frac{1}{2\rho}\sqrt{(\rho^2 - 1 + d^2)^2 - 4d^2} \right]$$

Mareno[6] gives

$$Z_o = 60 \cosh^{-1}\left[\frac{b}{2a}(1-\epsilon^2) + \frac{a}{2b} \right]$$

for an eccentric line. See figure 7.

Identity of these two equations is proved in Appendix III.

For this particular case, a characteristic impedance of fifty ohms is designed so the values of d, ρ, ϵ are restricted.

Mareno's characteristic impedance equation was first solved for the various values of ρ and ϵ which made

$$Z_o = 50\ \Omega$$

and a curve of ρ vs ϵ was plotted. See figure 7.

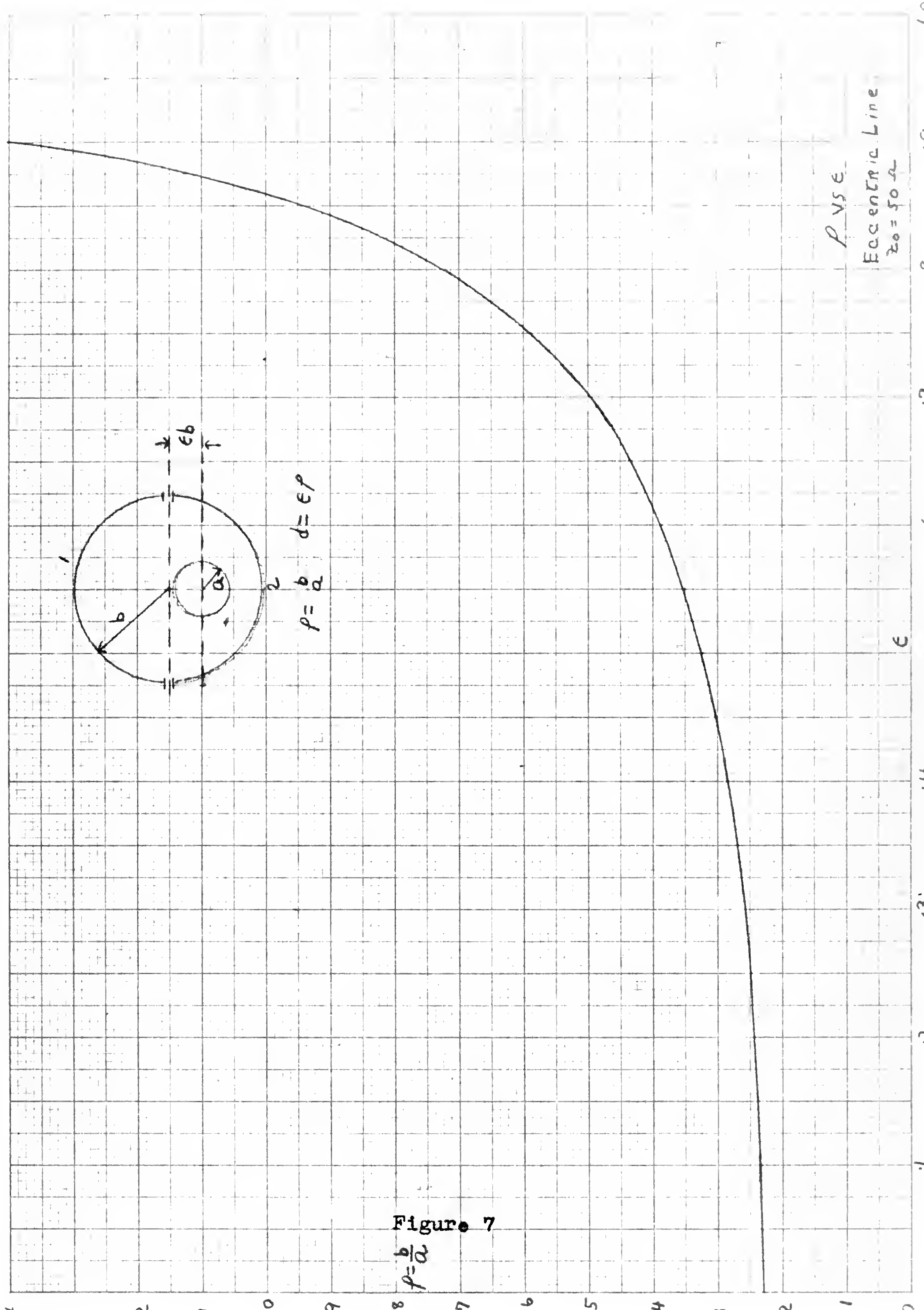
From
$$d = \epsilon \rho$$

the curve of ρ vs d resulted. See figure 8.

Knowing d, ϵ, ρ, $\frac{C_1}{C_T}$ and $\frac{C_2}{C_T}$ were then plotted vs d. See figure 9.

4. Slot Effect

So far, the effect of the slot on the coaxial line has been neglected. The current in a coaxial line propagating in the normal TEM mode is totally axial, so the total effect of the slot is to slightly increase the characteristic impedance by decreasing the capacitance.



ρ vs ε
Eccentric Line
$Z_0 = 50\,\Omega$
εb
b
a
1
2
$\rho = \frac{b}{a}$ $d = \epsilon\rho$
ε
$\rho = \frac{b}{a}$
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14

Figure 7

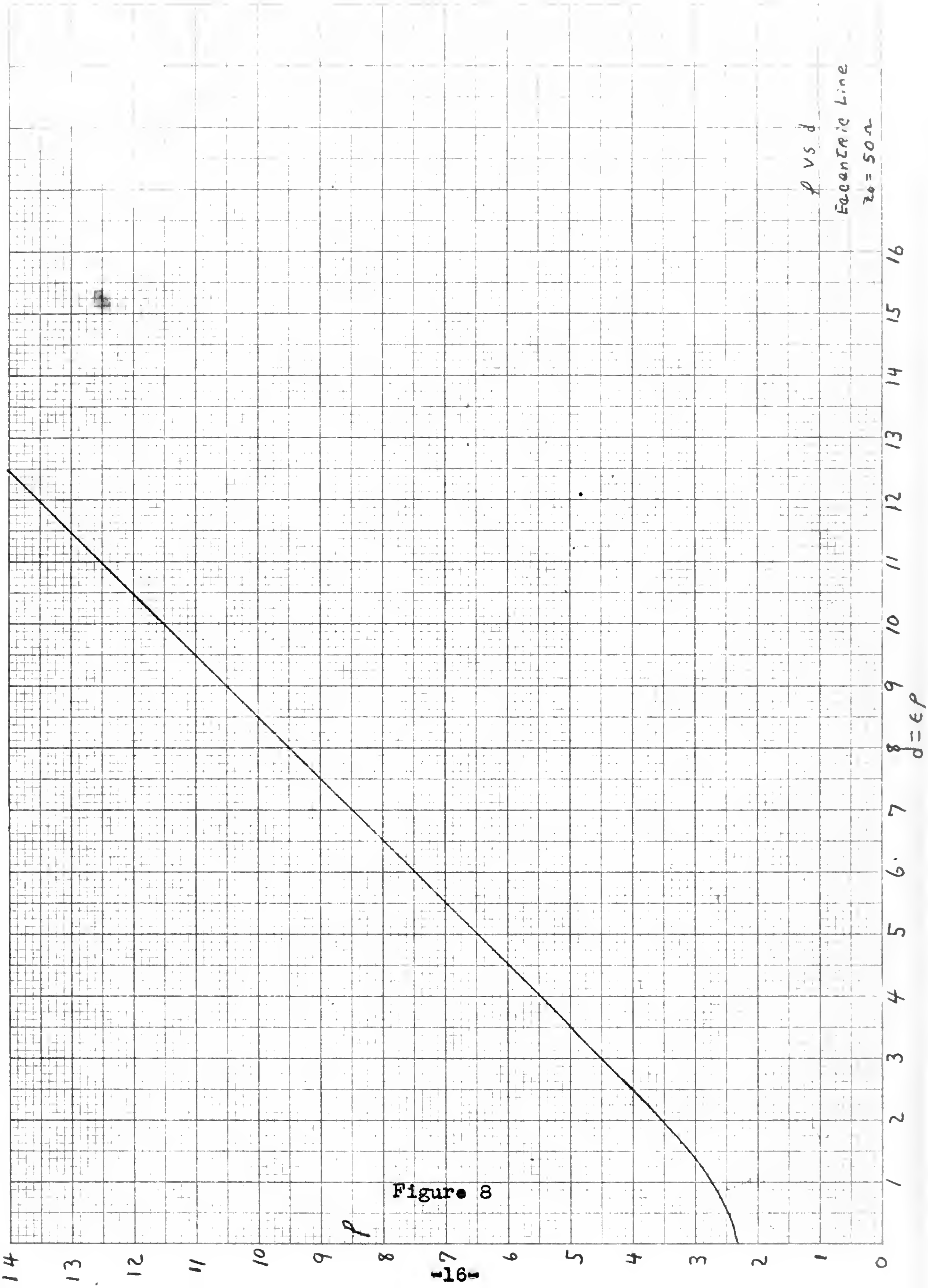
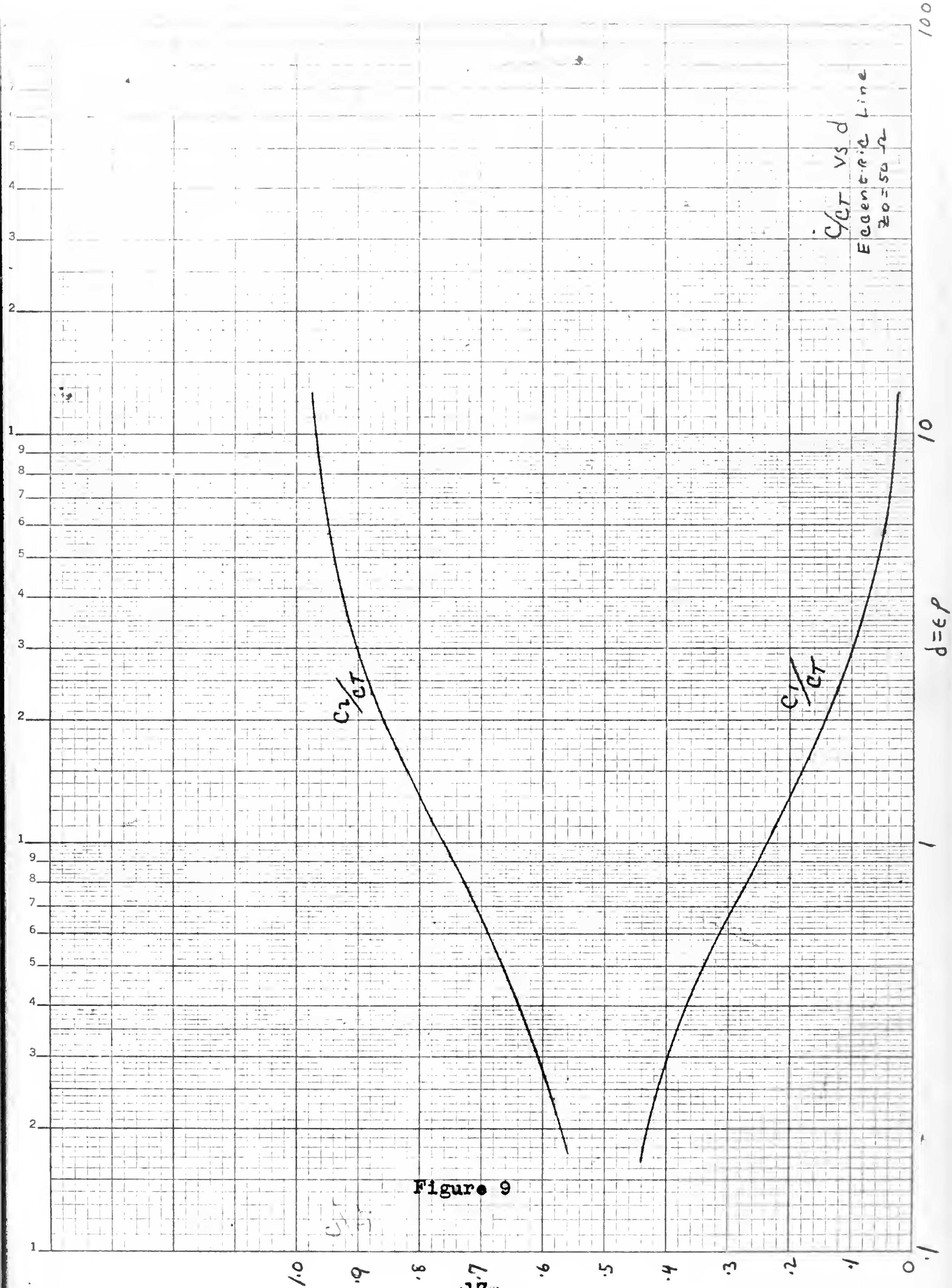


Figure 8



C/CT vs d
Eccentric Line
Z0=50 Ω
d=ερ
C1/CT
C'/CT
1.0 .9 .8 .7 .6 .5 .4 .3 .2 .1 0
.1
1
10
100

Figure 9

From Montgomery[7]

$$\frac{\Delta Z_o}{Z_o} = \frac{1}{4\pi^2} \cdot \frac{\phi'}{\beta^2-\alpha^2}$$

Where ϕ' is the slot width in the w-plane. From Appendix IV, it is seen that the exact expression for the slot angle in the w-plane is

$$\theta' = \tan^{-1} \frac{\theta(\rho^2-1+d^2)\sqrt{(1-\frac{\theta^2}{4})\left[(\rho^2-1-d^2)^2-4d^2\right]}}{4\rho^2 d^2 - \frac{\theta^2}{4}(\rho^2-1+d^2)^2 + (1-\frac{\theta^2}{4})\left[(\rho^2-1-d^2)^2-4d^2\right]}$$

but if the approximation that the angle θ subtended by the slot in the z-plane is small is made, then

$$\frac{\theta^2}{4} \simeq 0 \qquad \text{and} \qquad \tan\theta' \simeq \theta'$$

If θ' is the angle subtended in the w-plane, then

$$\theta' = \frac{\theta\sqrt{(\rho^2-1-d^2)-4d^2}}{\rho^2-1+d^2}$$

If

$$\rho' = \frac{\beta}{\alpha}$$

and it is known that the length of arc is the subtended angle multiplied by the radius, so

$$\phi' = \beta\theta'$$

and

$$\Delta Z_o = \frac{Z_o\,\rho'^2\,\theta'^2}{4\pi^2(\rho'^2-1)}$$

For the particular case used in the design, the increase in characteristic impedance for a slot width of .125 inches is .265 ohms which is negligible.

CHAPTER III

DESIGN OF AN ECCENTRIC SLOTTED BALUN

1. Use of Design Curves

The scales of the curves for $\frac{C}{C_t}$ vs d, φ vs d, and φ vs ϵ, were expanded so that normal values could be read easily. See figures 10, 11 and 12.

In this case, it was determined that it was necessary to transform a 300 ohm balanced load to a 50 ohm unbalanced load. This obviously calls for a transformation ratio of six to one, or

$$\frac{1}{a'^2} = 6$$

and

$$a' = .408$$

It is therefore desired

$$C_1 = .408\ C_t$$

From figure 10, it is seen that for

$$\frac{C_1}{C_t} = .408, \quad d = .275$$

From figure 11,

For $d = .275, \quad \varphi = 2.34$

From figure 12,

For $\varphi = 2.34, \quad \epsilon = .1175$

As a check, use

$$d = \epsilon\varphi = .1175 \times 2.34 = .275$$

which checks.

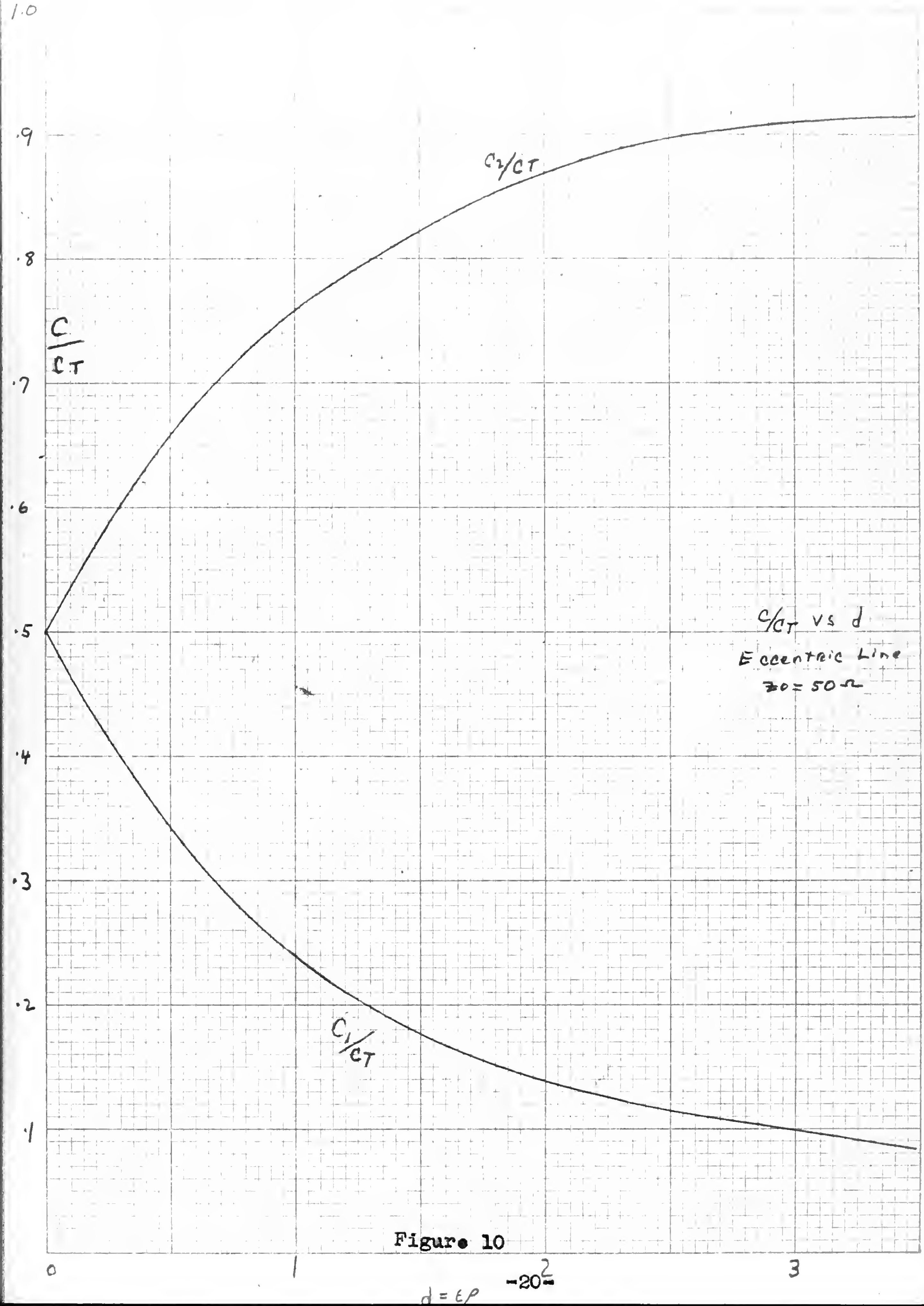

1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
C/CT
C2/CT
C1/CT
C/CT vs d
Eccentric Line
Zo = 50 Ω
0
1
2
3
d = ερ

Figure 10

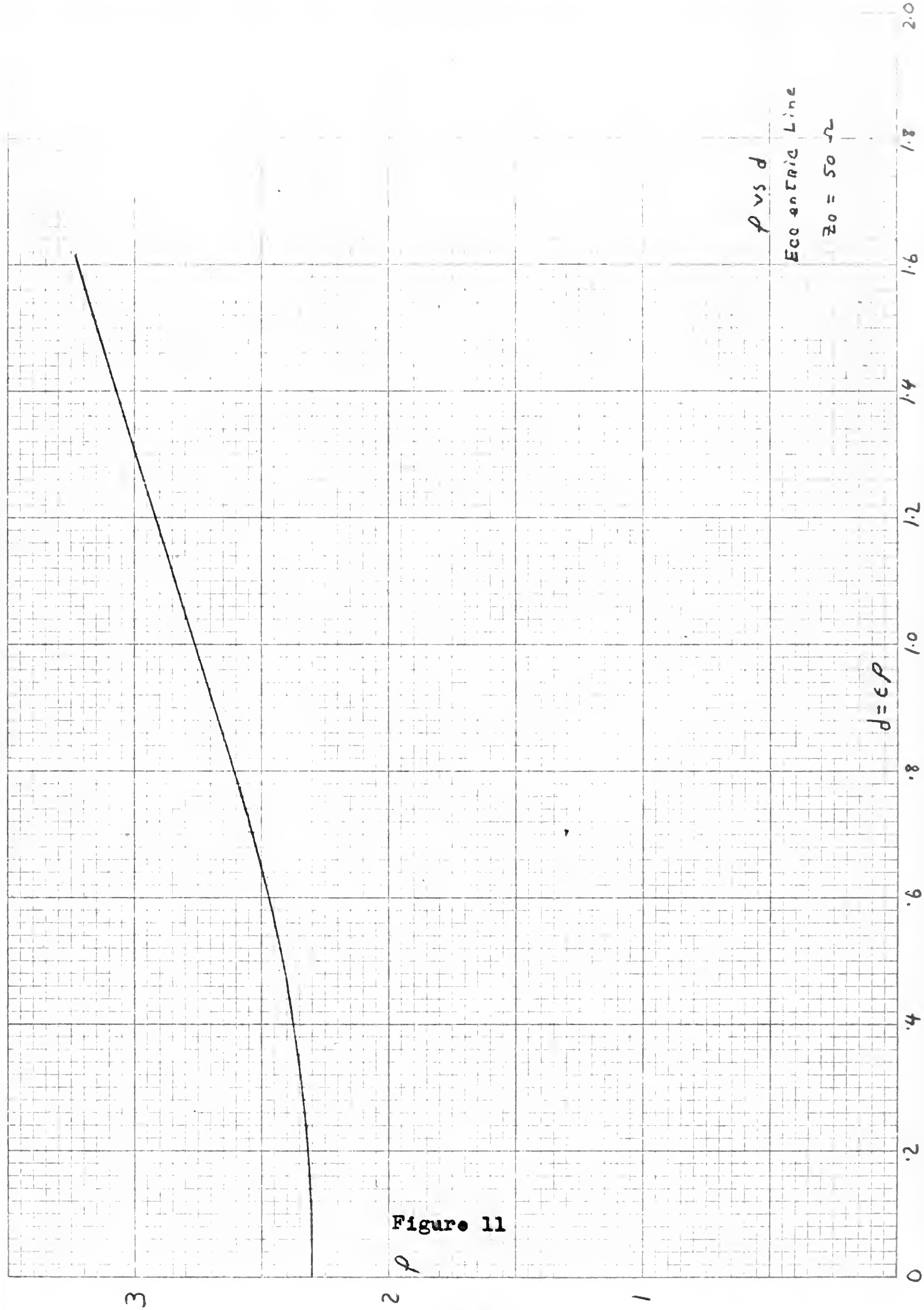


Figure 11

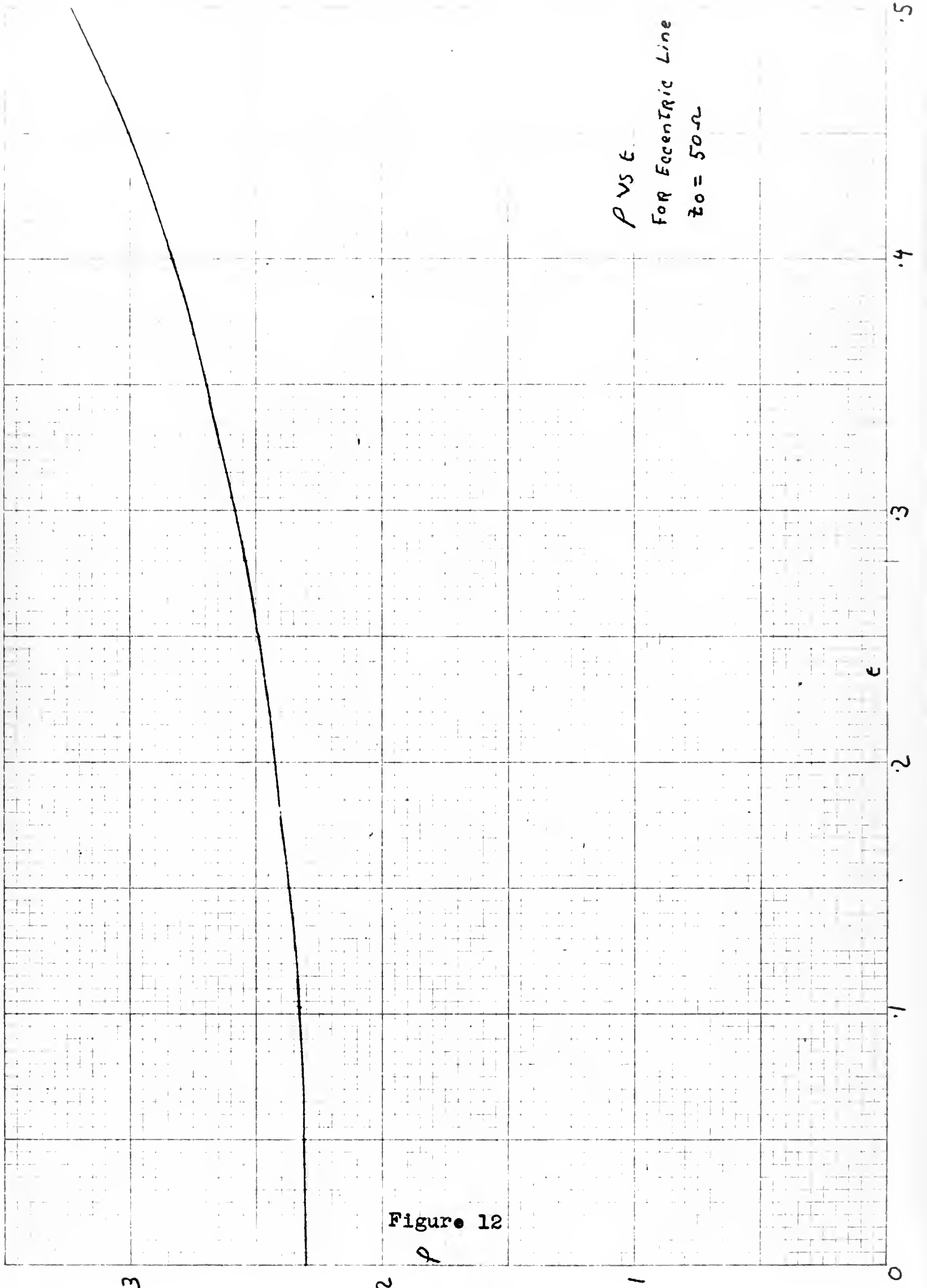


Figure 12

It was decided to attach the experimental model to the Hewlett Packard 805A slotted line. This required a center conductor of radius .175 inches. Then

$$a = .175''$$
$$b = .4095''$$
$$\epsilon b = .0481''$$

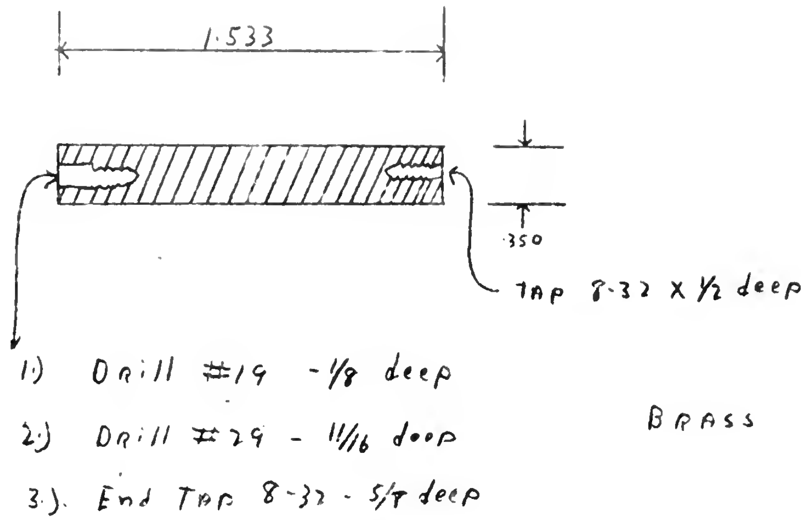
The slotted section of the balun will have an impedance transformation of six to one and a characteristic impedance of fifty ohms if the following parameters are used:

Inner diameter of outer conductor	.819 in.
Diameter of inner conductor	.350 in.
Displacement	.048 in.

The remainder of the design consists of choosing the method of offsetting the conductors. This will vary for the particular case, but it should be remembered that the characteristic impedance of the connecting system should be preserved at fifty ohms with as little discontinuity as possible.

In this case, it was decided to place the balun directly on the load end of the Hewlett Packard 805A slotted line. To do this, the type N connector and taper were removed from the slotted line. The characteristic impedance was preserved by making the normal connector taper housing a fifty ohm coaxial line. It was decided that the outer conductor should be displaced, so an offset coupler was designed to go from the 805A to the balun with the proper displacement. See figure 15. This will give a slight discontinuity, but it can be compensated in the final design.

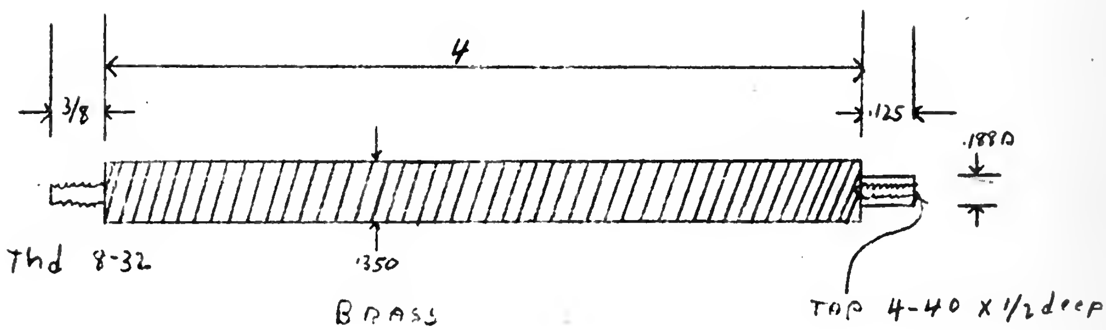
For the purpose of making the balun versatile and adaptable to any future changes, the center conductor was made in two sections. The load end was tapped so the short circuit could be made positively. See figures 13 and 14.



Center Conductor Part I

Eccentric Slotted Balun

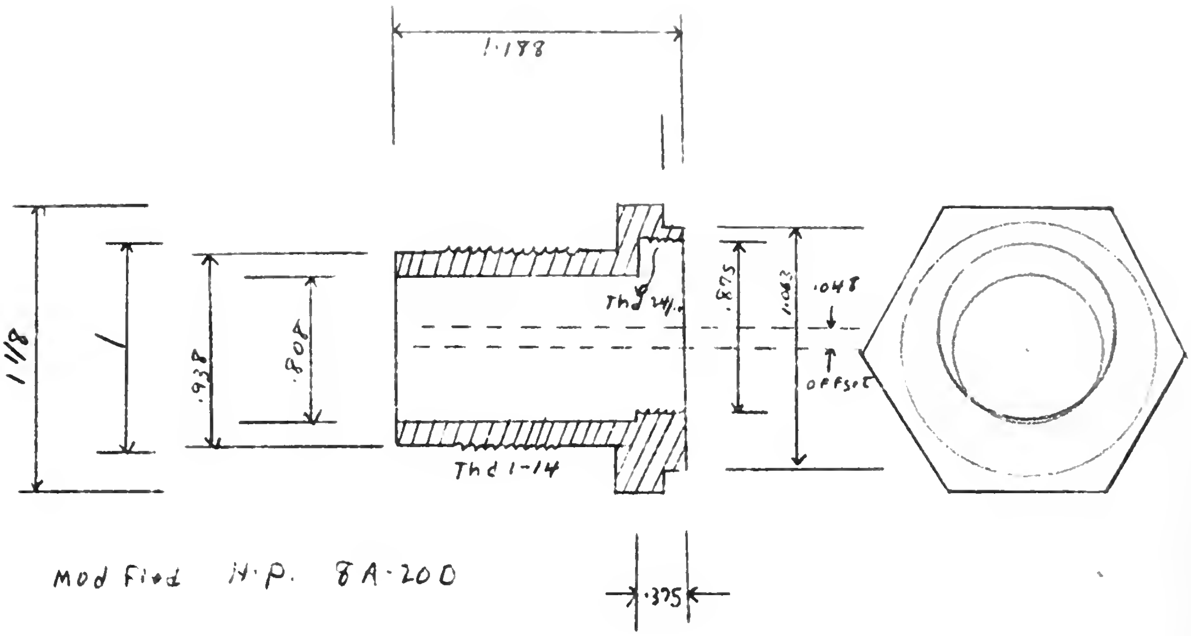
Figure 13



Center Conductor - Part II

Eccentric Slotted Balun

Figure 14



Modified H.P. 8A-200

OFFset Coupler

Eccentric slotted Balun

Figure 15

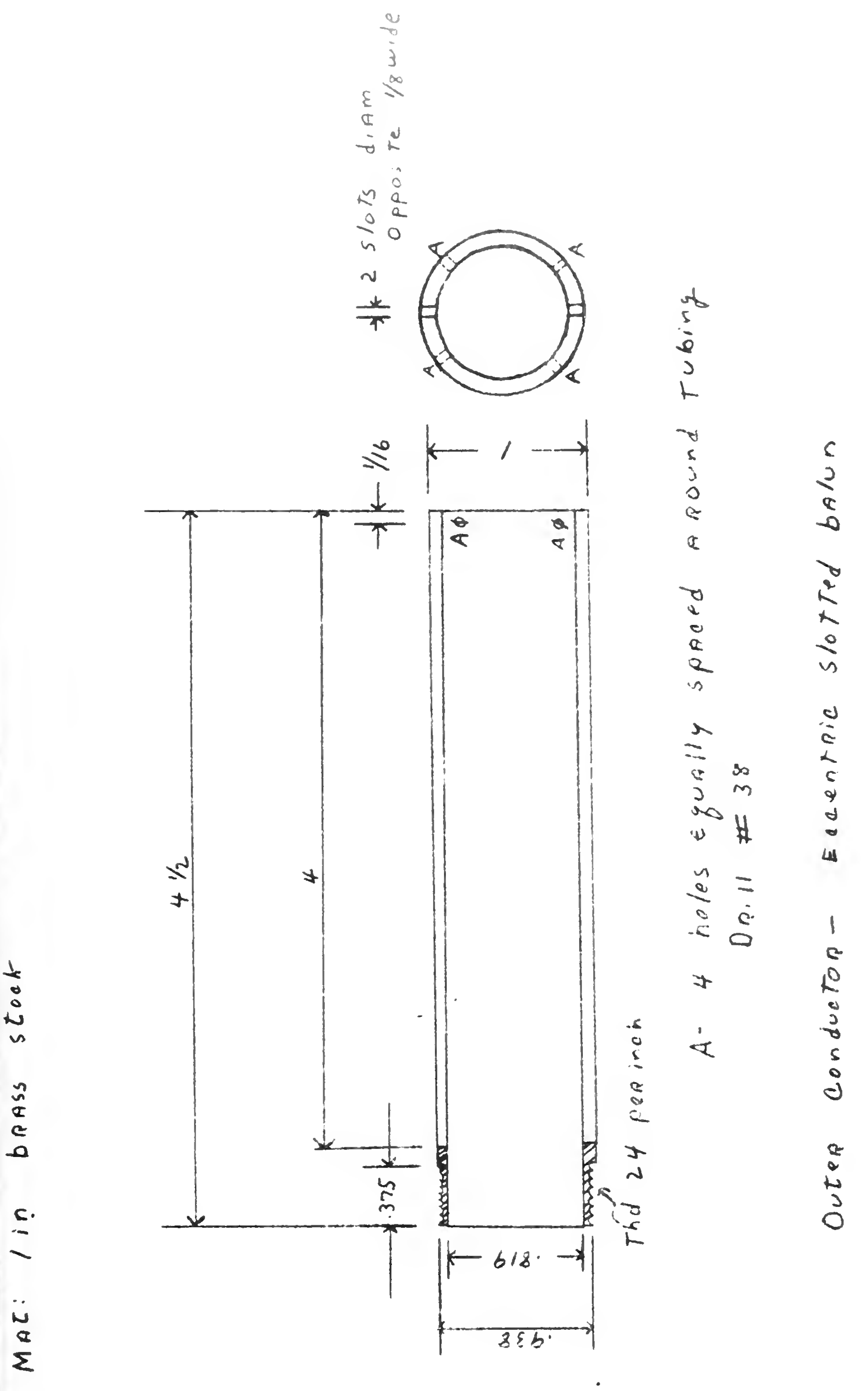
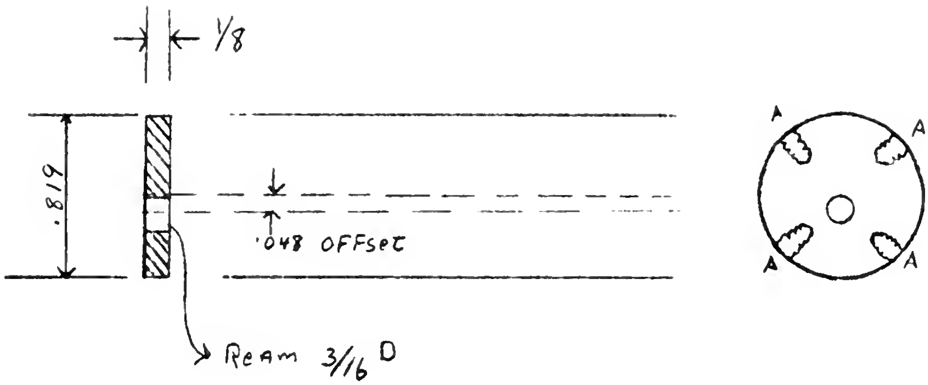

Mat: 1 in. brass stock
4 1/2
4
.375
1/16
.918
.938
Thd 24 per inch
AØ
AØ
1
A
A
A
A
2 slots diam opposite 1/8 wide
A- 4 holes equally spaced around Tubing
Drill #38
Outer Conductor - Eccentric slotted balun

Figure 16

Mat: Teflon



A- 4 holes equally spaced around disc
Tap 2-56 x 3/16 deep

End Support - Eccentric Slotted Balun

Figure 17

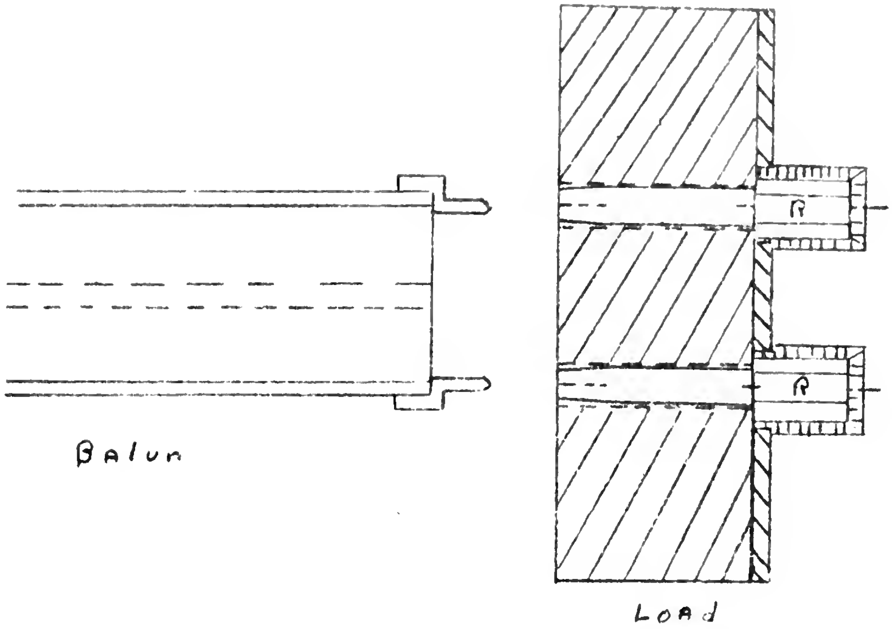
The outer conductor was reamed from one inch brass stock with 1/8 inch slots. By substituting into the equations of Appendix IV the increase in characteristic impedance for the two slots is found to be .265 ohms. This corresponds to a VSWR of 1.005 which is certainly negligible. The slots were only four inches long because this is an experimental model and a broad tuning range is not necessary to prove the theory. See figure 16.

A teflon end spacer was used to hold the outer conductor in its proper shape and to provide the proper spacing for the center conductor. Four holes were tapped for screws which came through the outer conductors, and the spacing hole was drilled with the proper displacement. See figure 17.

All materials were brass, so solder connections for the short circuit and the load could be used. The short circuit was a pie shaped piece of thin brass plate which was screwed to the center conductor and soldered to the closer half of the outer conductor.

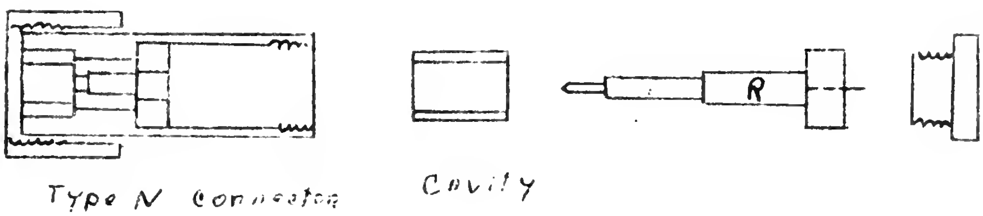
A balanced load was constructed using two 100 ohm Allen Bradley $\frac{1}{2}$ watt resistors placed in cylindrical brass cavities which were soldered on a three inch diameter ground plane. This made one end of each resistor terminate in a common ground. For connection to the balun, two type N female inner conductors were soldered to the resistors. The connectors were then mounted in a one inch thick piece of polystryrene three inches in diameter for support. A type N male inner conductor was soldered to each half of the balun outer conductor. Thus, the load could be easily interchanged with a short circuit for measurement purposes. See Figure 18.

The size of the above cavities was determined by trial and error. A type N male inner conductor was soldered to one end of a 100 ohm resistor, and a cylindrical brass shorting plug was soldered to the other. This



Balanced Load

Figure 18



Coaxial Load

Figure 19

combination was then placed in an UG/21B type N male connector forming a coaxial load. See Figure 19. The size of the cavity was varied with long cylindrical brass washers until a relatively constant and approximately resistive impedance was measured over the band of 450-1000 mcs. on the H.P. 805A slotted line.

In order to have a comparison of the eccentric slotted balun with something else, the concentric slotted balun was constructed. This was constructed in the same manner, and the same center conductor was used. The only difference was that that the inner diameter of the outer conductor was .809 inches, a straight coupler was used, and the end spacer had an alignment hole in the center. All parts were made interchangeable with the eccentric balun.

The balun was made large on the first trial to get away from close machining tolerances. For a balun of this size, the offset was small, less than a sixteenth of an inch, but for a smaller diameter balun, the displacement is much less and tolerances must be within one ten thousandth of an inch. This seemed impractical for the first model, so the larger was constructed.

CHAPTER IV

EXPERIMENTAL RESULTS

1. General Procedure

As mentioned in Chapter III, the balun was directly attached to the Hewlett Packard 805A slotted line. The H.P. Standing wave detector 415AR was used to measure VSWR. The H.P. 610A UHF signal generator was square wave modulated at a 1000 cycle rate and used as a source to feed the slotted line. See figure 20.

Impedance measurements were taken and plotted on Smith charts, thus the plotted impedances are normalized for the characteristic impedance of the slotted line, 50 ohms. The frequency range used was 450 - 1000 mcs. in all cases. At each setting of the signal generator, the following readings were taken:

a. VSWR

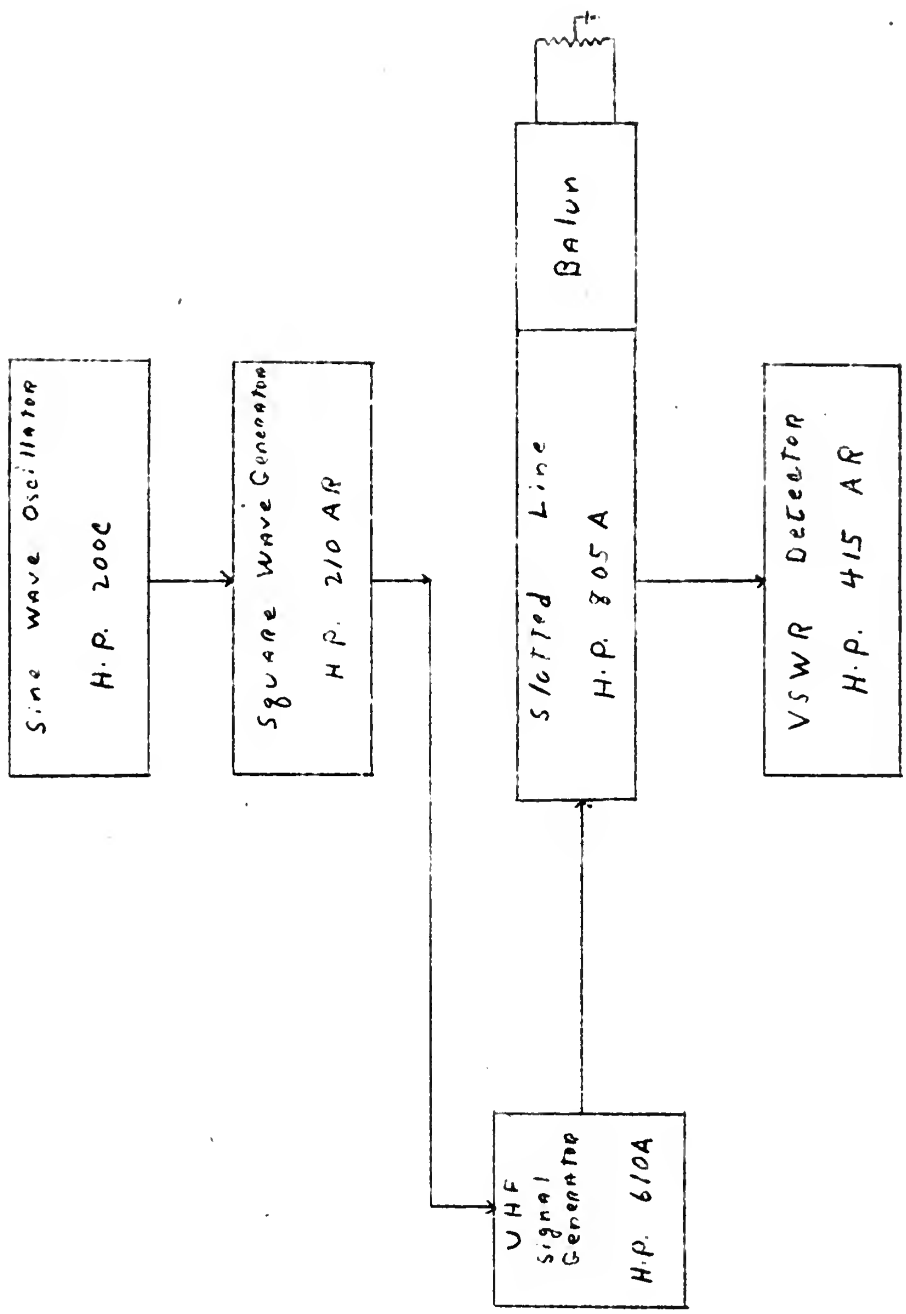
b. Positions of two successive minima with short

c. Positions of two successive minima with load

At the lower values of VSWR, the minima with the load attached were difficult to determine, so the half wavelength was checked with both the load and short circuit attached for greater accuracy. The above readings were taken on each set of impedance measurements rather than trust the accuracy of being able to reset a given frequency.

As an example of the calculations involved, take the 800 mc. position on Figure 21. On the scale of the slotted line the following readings were taken:

Short circuit minima	61.5	247.5



Experimental Set-up

Figure 20

Load minima	43.8	229.5
VSWR	2.5	

The average wavelength was 371.7 mm.

The average displacement of load minimum from short minimum was 17.85.

As can be seen, the load minimum moved to the left, towards the generator. The short circuit minimum corresponds to the true position of the load, but removed from it by an integral number of half wavelengths. This means that the load minimum lies a ratio of distance between minima for the shorted and loaded cases to the total wavelength, or .0481 wavelengths, toward the generator. To find the load impedance, move .0481 wavelengths towards the load, or counter clockwise on the Smith chart.

To plot this point, first move .0481 wavelengths towards the load, and plot a point at this angle a distance corresponding to a VSWR of 2.5. See Figure 21.

It will be noticed that the impedance plot approximates a circle. This would also be true if the load were the tank circuit illustrated in Figure 4. The balun was untuned for this set of measurements, and the load is resistive at 710 mcs. with a VSWR of 1.65.

A similar impedance plot was taken with the same load using the concentric balun. This also indicated a resistive load at 710 mcs. with a VSWR of 1.16. See Figure 22.

There were no means available to measure the balanced load at this frequency. The only possible check was to physically ground the center point of the two resistors to check the balance. When this was done no deflection of the 415 AR was noticeable, proving that the load was balanced.

Impedance characteristics
Eccentric slotted Balun II
For Load - 200 Ω
⊙ Measured

IMPEDANCE OR ADMITTANCE COORDINATES

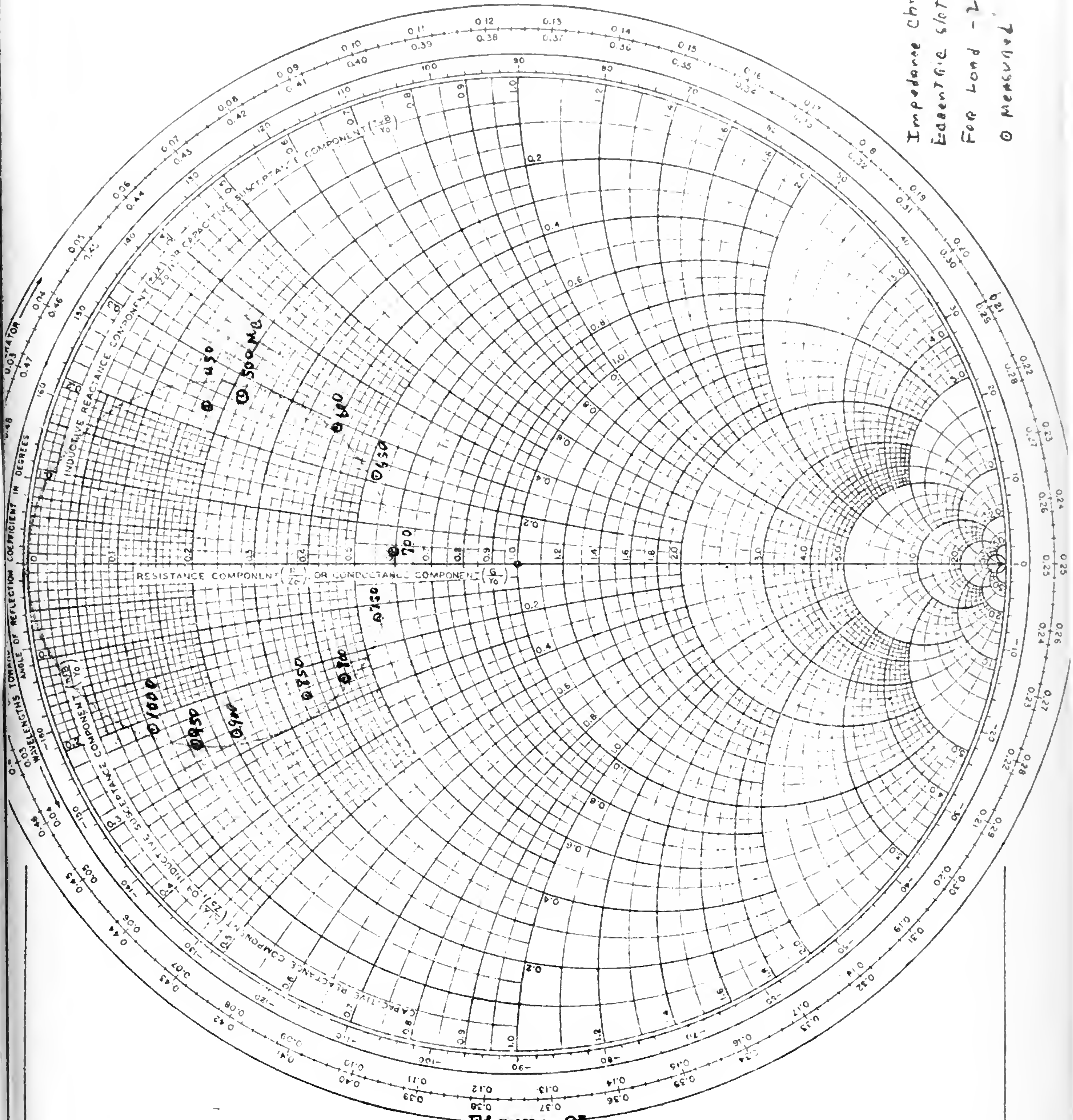
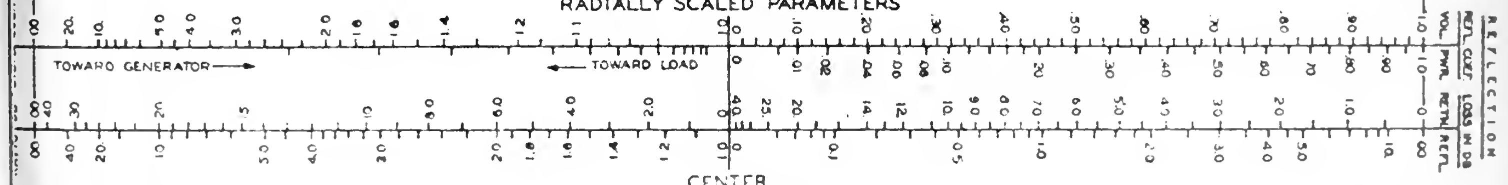


Figure 21

RADIALLY SCALED PARAMETERS



IMPEDANCE OR ADMITTANCE COORDINATES

Concentric Balun Load Characteristic For 200Ω Balanced Load

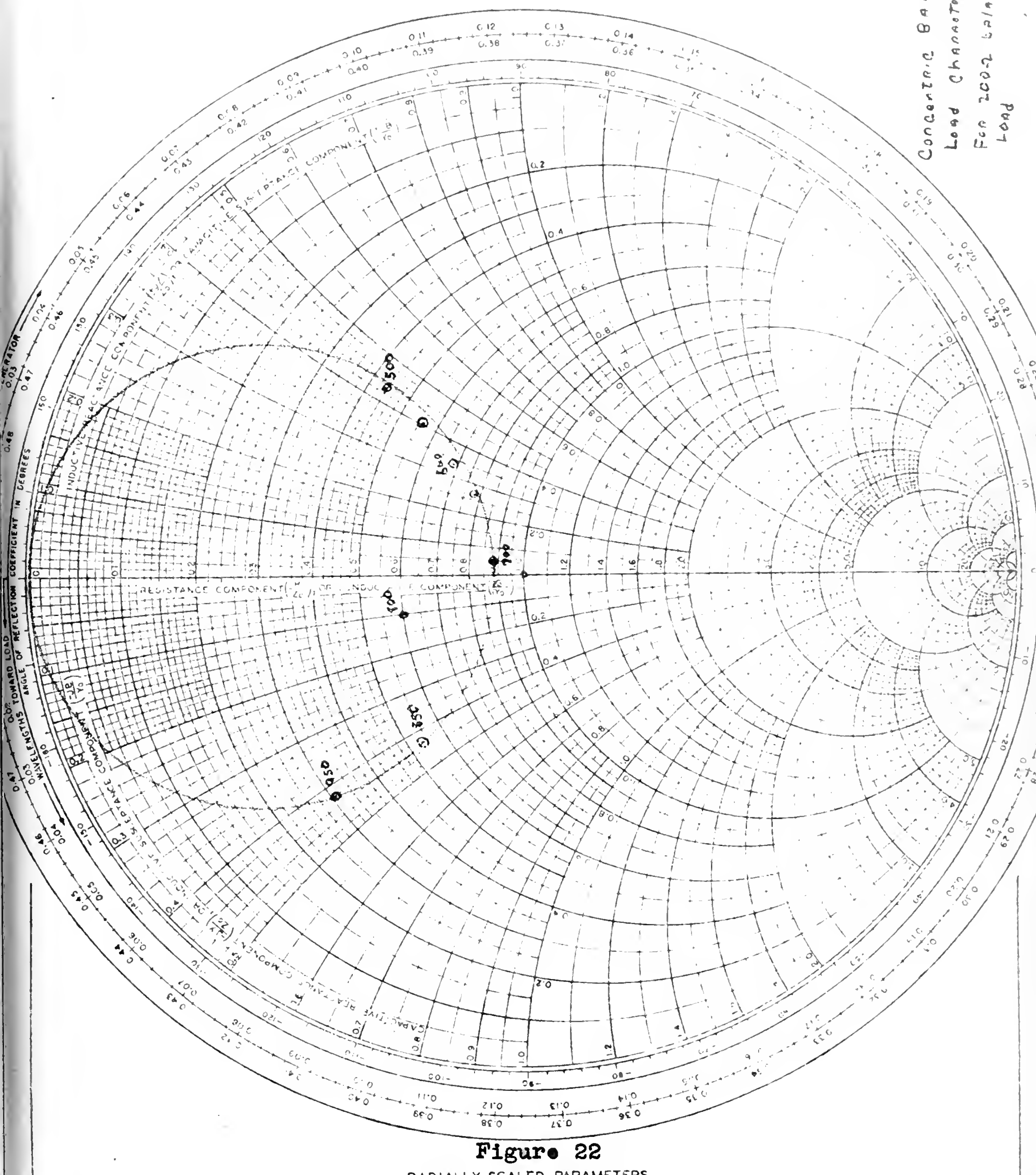
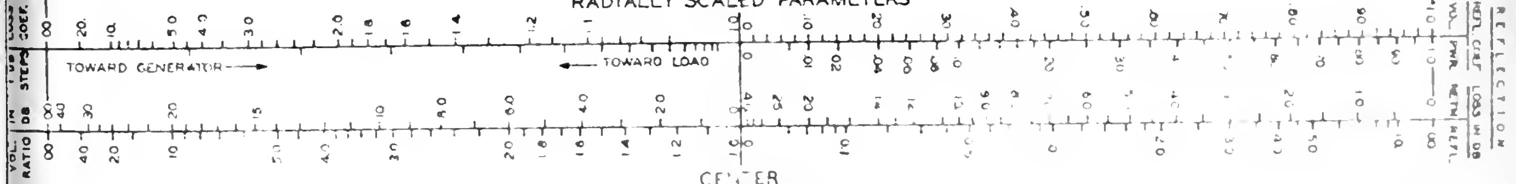


Figure 22

RADIALLY SCALED PARAMETERS



DR. | TITLE | TELEPHONE LABORATORIES, INC.
ENG.

Impedance characteristics Eccentric slotted Balun II For Load - 100' of 300Ω twin lead

IMPEDANCE OR ADMITTANCE COORDINATES

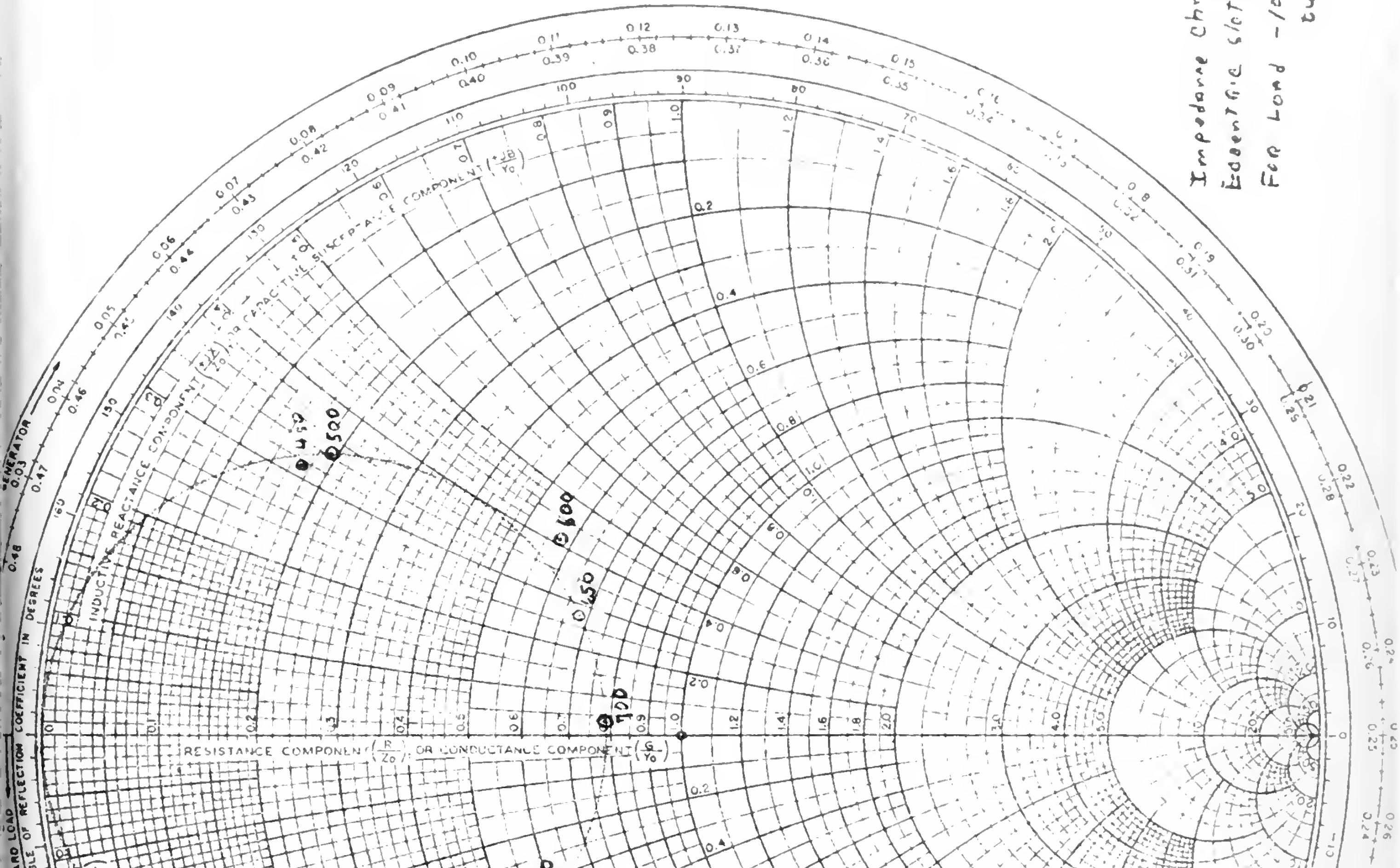
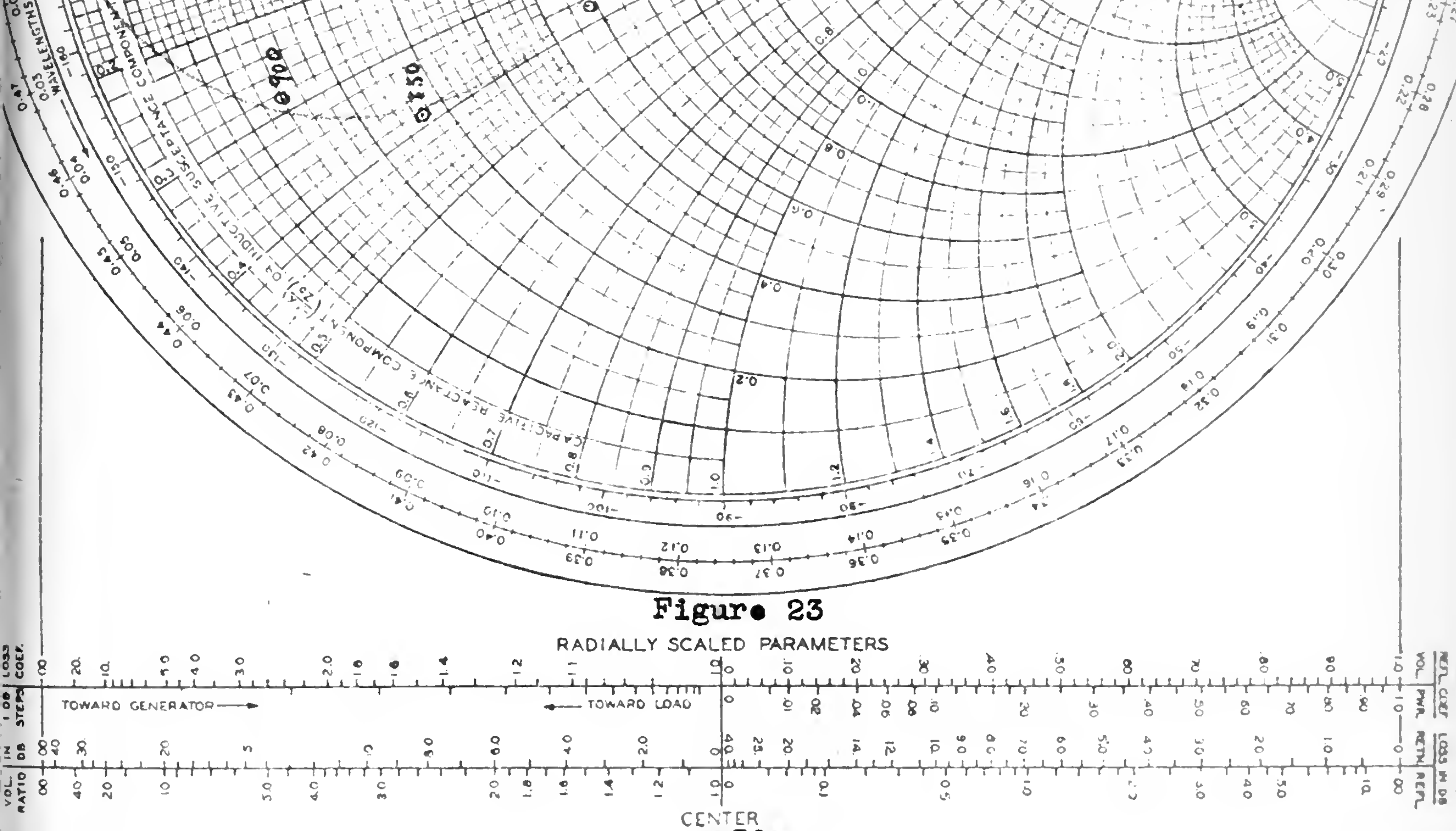


Figure 23

RADIALLY SCALED PARAMETERS



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In an attempt to find a known balanced load, 100 feet of 300 ohm twin lead was soldered to the balun. The line was terminated in an ordinary 300 ohm Allen Bradley resistor, but a flat line was not obtained. See Figure 23.

2. Evaluation

Without knowing the exact value of the load, some assumptions must be made. If it is first assumed that the concentric balun is perfect and has an impedance transformation of 4:1, then the load impedance at 710 mcs. is

$$Z_L = 4 \times \frac{50}{1.16} = 172.4\ \Omega$$

If the load impedance is now calculated assuming a 6:1 impedance transformation for the eccentric slotted balun, the same answer should be obtained:

$$Z_L = 6 \times \frac{50}{1.65} = 181.5\ \Omega$$

Using the concentric slotted balun as a reference, this will give an error of 5.3% for the eccentric balun.

If no assumptions are made it is seen that the ratio of the impedance transformations for the concentric and eccentric baluns should be 4/6 for the same load. The error in this ratio is only 3%. This proves the theory without assumptions, but tells nothing about the transformation ratios.

If time had permitted, a satisfactory balanced load would have been developed to obtain the impedance transformations directly.

The eccentric balun was then tuned with a shorting strip around the outer conductor. The maximum variation was from a VSWR of 1.65 to 1.70, (Figure 24) representing a 3.03% error which could easily be accounted for in the load.

Even though the balun was unshielded, no variations in readings were noticed unless the slots themselves were touched. During the tuning process,

the slots could be touched behind (generator end) the shorting strips without any effect, which indicates that the assumption of shielding being perfect was correct.

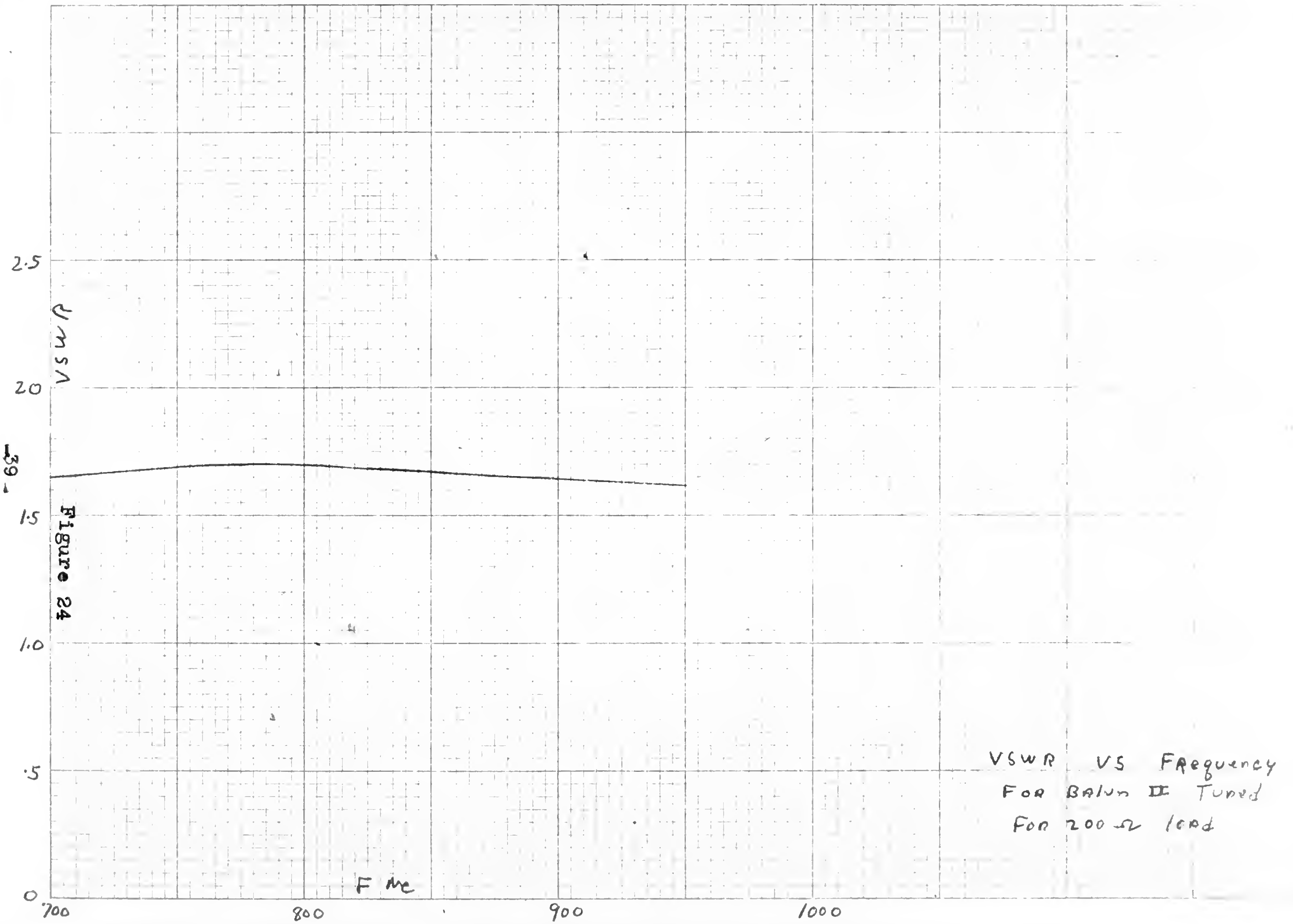
VSWR VS Frequency
For Balun II Tuned
For 200 Ω load
VSWR
F Mc
2.5
2.0
1.5
1.0
.5
0
700
800
900
1000

Figure 24

CHAPTER V

CONCLUSION

Even though a known balanced load was not available, the method of comparing input impedances of the concentric and eccentric baluns is sufficient to prove the theory of the eccentric slotted balun. This method showed an error of 3% from the theoretical value. Even if it is assumed that the transformation ratios in the two cases are respectively 4:1 and 6:1, an error of only 5.3% resulted.

The balun would be much better with a shield, but as long as the slots are free, the balun acts as if it were well shielded.

The eccentric balun is easily tuned with a simply constructed shorting strip around the outer conductor. If a shield were used, both the slot and shield lengths would have to be varied by shorting strips or fingers, but this could be done in one operation.

This was the initial stage of the development. The next step would be to construct a balanced load so that the impedance transformation would be known. At the same time, a balance comparator[13] should be constructed in order to check balance efficiency.

Following this, a shield should be constructed with a means devised of tuning both shield and slots simultaneously.

The physical size of the balun could be varied to meet any particular condition. For example, it would be ideal if it were constructed of such a size to fit a standard type N connector. This would necessitate close machining tolerances for the first model, but this could then be used to construct a mold. Future models for production purposes could then be cast.

The eccentric slotted balun is more difficult to construct than most coaxial baluns, but when it is completed, an efficient, accurate and easily tuned laboratory instrument results.

The design curves in Chapter III can be used to design baluns with impedance transformations, within practical physical limits, ranging from about two to twenty.

The balun uses are many. Among them are balanced UHF receiver alignments, balanced impedance measurements, and balanced antenna measurements. This balun is small, and HF or VHF antennas could be scaled down easily and measurements taken at UHF frequencies. It is an instrument which would prove valuable in any laboratory.

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APPENDIX I

THE CONFORMAL TRANSFORMATION

Because of the difficulty of deriving the capacitance in the eccentric line, the following conformal transformation was used to transform the eccentric case to the coaxial case.

If the reader refers to Figure 6, it is seen that the following symbols are used in connection with the transformation:

a - radius of inner conductor in z-plane

b - radius of outer conductor in z-plane

x_1 - center of inner conductor in z-plane

x_2 - center of outer conductor in z-plane

δ - distance between conductor centers in z-plane

ϕ - slot width in z-plane

α - radius of inner conductor in w-plane

β - radius of outer conductor in w-plane

ϕ' - slot width in w-plane

θ - angle subtended by slot at x_2 in z-plane

Θ - angle subtended by slot in w-plane

The transformation which will transform eccentric circles in the z-plane to concentric circles in the w-plane is

1) $$W = \frac{z-\rho}{z+\rho}$$

We can show that this is true by the inverse process. That is, by showing that two concentric circles in the w-plane transform into two eccentric circles in the z-plane.

If $|W| = c$, then $c = \left|\frac{z-\rho}{z+\rho}\right| = \left|\frac{x-\rho+jy}{x+\rho+jy}\right|$

$$c = \frac{\sqrt{(x-\rho)^2+y^2}}{\sqrt{(x+\rho)^2+y^2}}$$

$$x^2 - 2\rho x + y^2 = c^2\left[x^2 + 2\rho x + \rho^2 + y^2\right]$$

2) $$\left[x - \rho\left(\frac{1+c^2}{1-c^2}\right)\right]^2 + y^2 = \left(\frac{2\rho c}{1-c^2}\right)^2$$

This is the equation of a circle, so a circle of radius c in the w-plane will map into a circle in the z-plane with

center at $$x_c = \rho\left(\frac{1+c^2}{1-c^2}\right)$$

and radius $$R = \frac{2\rho c}{1-c^2}$$

Thus two concentric circles in the w-plane with radii α and β will map into two eccentric circles in the z-plane with radii a and b and centers at x_1 and x_2.

We know that the right hand semicircle of radius b will be mapped into an arc A-A' in the w-plane. This is a symmetrical transformation, so the angle subtended will be double the angle subtended in the upper segment.

IF $z = x_2 + jb = A$

$$w = \frac{x_2 - p + jb}{x_2 + p + jb} \qquad \text{let } \rho = \frac{b}{a}$$

3) $$w = \frac{x_2/a - p/a + j\rho}{x_2/a + p/a + j\rho}$$

FROM 2) NOTE THAT:

$$x_c^2 - \rho^2 = \frac{4\rho^2 c^2}{(1-c^2)^2} = R^2$$

4) $$\therefore \quad x_1^2 - p^2 = a^2$$

5) $$x_2^2 - p^2 = b^2$$

IT FOLLOWS THAT:

$$x_1^2 - x_2^2 = a^2 - b^2$$

$$x_2 - x_1 = \delta$$

SO $$\frac{x_1^2 - x_2^2}{x_2 - x_1} = \frac{b^2 - a^2}{\delta} = x_1 + x_2$$

$$x_2 = \frac{b^2 - a^2 + \delta^2}{2\delta}$$

$$x_1 = \frac{b^2 - a^2 - \delta^2}{2\delta}$$

let $d = \delta/a$

6) $$\frac{x_2}{a} = \frac{b^2/a^2 - 1 + \delta^2/a^2}{2\delta/a} = \frac{\rho^2 - 1 + d^2}{2d}$$

7) $$\frac{x_1}{a} = \frac{\rho^2 - 1 - d^2}{2d}$$

FROM 4) $$p = \sqrt{x_1^2 - a^2}$$

$$p/a = \left(\frac{a^2\left(\frac{\rho^2 - 1 - d^2}{4d^2}\right)^2 - a^2}{a^2}\right)^{1/2}$$

8) $$p/a = \sqrt{\frac{[\rho^2 - (1-d)^2][\rho^2 - (1+d^2)^2]}{4d^2}}$$

3a) From 3), 6), 8) $W = \dfrac{\dfrac{\rho^2-1+d^2}{2d} - \sqrt{\dfrac{[\rho^2-(1-d)^2][\rho^2-(1+d)^2]}{4d^2}} + j\rho}{\dfrac{\rho^2-1+d^2}{2d} + \sqrt{\dfrac{[\rho^2-(1-d)^2][\rho^2-(1+d)^2]}{4d^2}} + j\rho}$

$$\therefore \text{ARG } W = \tan^{-1} \frac{\rho}{\dfrac{\rho^2-1+d^2}{2d} - \sqrt{\dfrac{[\rho^2-(1-d)^2][\rho^2-(1+d)^2]}{4d^2}}}$$

$$-\tan^{-1} \frac{\rho}{\dfrac{\rho^2-1+d^2}{2d} + \sqrt{\dfrac{[\rho^2-(1-d)^2][\rho^2-(1+d)^2]}{4d^2}}}$$

$$\text{ARG } W = \tan^{-1} \frac{2\rho d}{\rho^2-1+d^2 - \sqrt{(\rho^2-1-d^2)^2 - 4d^2}}$$

$$-\tan^{-1} \frac{2\rho d}{\rho^2-1+d^2 + \sqrt{(\rho^2-1-d^2)^2 - 4d^2}}$$

Now: $\tan^{-1} \dfrac{a}{b-c} - \tan^{-1} \dfrac{a}{b+c} = \tan^{-1} \dfrac{2ac}{a^2+b^2-c^2}$

It follows that:

9) $\text{Tan ARG } W = \dfrac{1}{2\rho d} \sqrt{(\rho^2-1-d^2)^2 - 4d^2}$

APPENDIX II

CHARACTERISTIC IMPEDANCE OF ECCENTRIC LINE

Continuing the use of conformal transformations, the characteristic impedance of the eccentric line is now derived.

10) $$Z_0 = 60 \ln \frac{B}{\alpha}$$

11) From 2) $$x_1 = \rho \frac{1+\alpha^2}{1-\alpha^2} \qquad a = \frac{2\rho\alpha}{1-\alpha^2}$$

12) $$x_2 = \rho \frac{1+B^2}{1-B^2} \qquad b = \frac{2\rho B}{1-B^2}$$

$$d = \frac{\delta}{a} = \frac{x_2}{a} - \frac{x_1}{a} = \left[\frac{1+B^2}{1-B^2} - \frac{1+\alpha^2}{1-\alpha^2}\right] \frac{1-\alpha^2}{2\alpha}$$

OR $$d = \left(\frac{1+B^2}{1-B^2}\right)\left(\frac{1-\alpha^2}{2\alpha}\right) - \frac{1+\alpha^2}{2\alpha}$$

Now $$\rho = \frac{b}{a} = \frac{B}{\alpha}\left(\frac{1-\alpha^2}{1-B^2}\right)$$

13) So that $$d = \rho\left(\frac{1+B^2}{2B}\right) - \frac{1+\alpha^2}{2\alpha}$$

From 10), 11) $$\rho = \frac{a(1-\alpha^2)}{2\alpha} = \frac{b(1-B^2)}{2B}$$

14) OR $$\rho\cdot\left(\frac{1-B^2}{2B}\right) - \left(\frac{1-\alpha^2}{2\alpha}\right) = 0$$

Taking the sum of 13) and 14)

15) $$\frac{\rho}{B} - \frac{1}{\alpha} = d$$

Taking the difference of 13) and 14)

16) $$\rho B - \alpha = d$$

Solving 15) and 16) for B

17) $$B = -\frac{1}{2\rho d}(1-d^2-\rho^2) \pm \frac{\sqrt{(1-d^2-\rho^2)^2 - 4\rho^2 d^2}}{2\rho d}$$

18) From 15) $$\frac{B}{\alpha} = \rho - Bd$$

19) $$\therefore \frac{B}{\alpha} = \rho + \frac{1}{2\rho}(1-d^2-\rho^2) \mp \frac{1}{2\rho}\sqrt{(1-d^2-\rho^2)^2 - 4\rho^2 d^2}$$

To check the proper sign in 19):

For $+$ $\quad \lim_{d \to 0} \frac{\beta}{\alpha} = \rho$

For $-$ $\quad \lim_{d \to 0} \frac{\beta}{\alpha} = \frac{1}{4\rho}$

$\therefore$ The + sign will be used

20) Substiting in 10):

$$Z_0 = 60 \ln \left[\rho - \frac{1}{2\rho}(\rho^2 - 1 + d^2) + \frac{1}{2\rho}\sqrt{(\rho^2 - 1 + d^2)^2 - 4\rho^2 d^2} \right]$$

APPENDIX III

IDENTITY OF CHARACTERISTIC IMPEDANCE EQUATIONS

To verify the derivation by means of conformal transformation, a well known equation for the characteristic impedance of an eccentric line was taken, and is now proved identical with the one derived.

21) $$Z_o = 60 \cosh^{-1}\left[\frac{b}{2a}(1-\epsilon^2) + \frac{a}{2b}\right]$$

Where

$$\frac{b}{a} = \rho \qquad \epsilon = \frac{ad}{b} = \frac{d}{\rho}$$

$$d = \frac{\epsilon b}{a} = \epsilon\rho$$

Now $$\cosh^{-1} u = \ln\left(u + \sqrt{u^2 - 1}\right)$$

$$u = \frac{\rho}{2} - \frac{d^2}{2\rho} + \frac{1}{2\rho}$$

$$u^2 = \frac{\rho^2}{4} - \frac{2d^2}{4} + \frac{2}{4} + \frac{d^4}{4\rho^2} - \frac{2}{4}\frac{d^2}{\rho^2} + \frac{1}{4\rho^2}$$

$$\therefore \cosh^{-1} u = \ln\left[\frac{\rho}{2} - \frac{d^2}{2\rho} + \frac{1}{2\rho} + \sqrt{\frac{\rho^2}{4} - \frac{2d^2}{4} + \frac{2}{4} - \frac{2d^2}{4\rho^2} + \frac{1}{4\rho^2} - 1}\right]$$

22) $$\cosh^{-1} u = \ln\left[\rho - \frac{1}{2\rho}(\rho^2 - 1 + d^2) + \frac{1}{2\rho}\sqrt{(\rho^2 - 1 + d^2)^2 - 4\rho^2 d^2}\right]$$

$\therefore$ Equations 21) and 20) are identical

APPENDIX IV

EFFECTIVE SLOT WIDTH IN THE W-PLANE

To show that the slot has a negligible effect on a coaxial transmission line, the exact and approximate expressions for the effective slot width in the w-plane are now derived in terms of design parameters.

The formula for the change in characteristic impedance is also derived in terms of the same parameters.

Effective slot width in the w-plane

We will first find the general expression for the transformation of a point P on the outer circle in the z-plane to a point in the w-plane for

$$23) \qquad P = x_2 + \gamma + j\sqrt{b^2 - \gamma^2}$$

where γ is positive to the right of x_2, and negative to the left of x_2. See figure .

$$24) \qquad W = \frac{x_2 + \gamma - \rho + j\sqrt{b^2 - \gamma^2}}{x_2 + \gamma + \rho + j\sqrt{b^2 - \gamma^2}}$$

$$W = \frac{\frac{x_2}{a} + \frac{\gamma}{a} - \frac{\rho}{a} + j\sqrt{\rho^2 - \frac{\gamma^2}{a^2}}}{\frac{x_2}{a} + \frac{\gamma}{a} + \frac{\rho}{a} + j\sqrt{\rho^2 - \frac{\gamma^2}{a^2}}}$$

from 3a)

$$25) \qquad W = \frac{\frac{\rho^2 - 1 + d^2}{2d} + \frac{\gamma}{a} - \sqrt{\frac{[\rho^2 - (1-d)^2][\rho^2 - (1+d)^2]}{4d^2}} + j\sqrt{\rho^2 - \frac{\gamma^2}{a^2}}}{\frac{\rho^2 - 1 + d^2}{2d} + \frac{\gamma}{a} + \sqrt{\frac{[\rho^2 - (1-d)^2][\rho^2 - (1+d)^2]}{4d^2}} + j\sqrt{\rho^2 - \frac{\gamma^2}{a^2}}}$$

and it follows that

$$26) \qquad \operatorname{Arg} W = \tan^{-1} \frac{2d\sqrt{\rho^2 - \frac{\gamma^2}{a^2}}}{(\rho^2 - 1 + d^2) + \frac{2d\gamma}{a} - \sqrt{(\rho^2 - 1 - d^2)^2 - 4d^2}}$$

$$- \tan^{-1} \frac{2d\sqrt{\rho^2 - \frac{\gamma^2}{a^2}}}{(\rho^2 - 1 + d^2) + \frac{2d\gamma}{a} + \sqrt{(\rho^2 - 1 - d^2)^2 - 4d^2}}$$

$$27) \text{ and } \qquad \operatorname{Arg} W = \tan^{-1} \frac{\sqrt{(\rho^2 - \frac{\gamma^2}{a^2})[(\rho^2 - 1 - d^2)^2 - 4d^2]}}{2\rho^2 d + \frac{\gamma}{a}(\rho^2 + d^2 - 1)}$$

For $\gamma = 0$, this reduces to 9).

If we let the slot width in the z-plane be ϕ, then with ϕ small $\gamma = \frac{\phi}{2}$. If θ is the angle subtended at x_2 by the slot, then

$$\theta = \frac{\phi}{b} \qquad \text{And} \qquad \frac{\gamma}{a} = \frac{\theta b}{2a} = \frac{\rho\phi}{2}$$

If we let θ' be the angle subtended by the slot in the w-plane, then

28) $$\theta' = \theta_2' - \theta_1'$$

where

$$\theta_1' = \tan^{-1} \frac{\rho \sqrt{(1-\frac{\theta^2}{4})[(\rho^2-1-d^2)^2 - 4d^2]}}{2\rho^2 d + \frac{\rho\theta}{2}(\rho^2-1+d^2)}$$

$$\theta_2' = \tan^{-1} \frac{\rho \sqrt{(1-\frac{\theta^2}{4})[(\rho^2-1-d^2)^2 - 4d^2]}}{2\rho^2 d - \frac{\rho\theta}{2}(\rho^2-1+d^2)}$$

From $$\tan^{-1}\frac{a}{b} - \tan^{-1}\frac{a}{c} = \tan^{-1}\frac{a(c-b)}{bc+a^2}$$

29) $$\theta' = \tan^{-1} \frac{\theta(\rho^2-1+d^2)\sqrt{(1-\frac{\theta^2}{4})[(\rho^2-1-d^2)^2-4d^2]}}{4\rho^2 d^2 - \frac{\theta^2}{4}(\rho^2-1+d^2)^2 + (1-\frac{\theta^2}{4})[(\rho^2-1-d^2)^2-4d^2]}$$

If we make the approximations that

$\theta < 1$ so that $\frac{\theta^2}{4} \ll 1$

and $\tan\theta' \simeq \theta'$

Then

30) $$\theta' = \frac{\theta(\rho^2-1+d^2)\sqrt{(\rho^2-1-d^2)^2-4d^2}}{4\rho^2 d^2 + (\rho^2-1-d^2)^2 - 4d^2}$$

or

31) $$\theta' = \frac{\theta\sqrt{(\rho^2-1-d^2)^2-4d^2}}{(\rho^2-1+d^2)}$$

32) $$\frac{\Delta z_0}{z_0} = \frac{1}{4\pi^2} \cdot \frac{\phi^2}{B^2 - \alpha^2}$$

$$= \frac{1}{4\pi^2} \cdot \frac{B^2 \theta'^2}{B^2 - \alpha^2}$$

$$= \frac{1}{4\pi^2} \cdot \frac{B^2}{\alpha^2} \cdot \frac{\theta'^2}{B^2/\alpha^2 - 1}$$

$\rho' = \frac{\beta}{\alpha} =$ Constant For z_0 Constant

33) Therefore $$\Delta z_0 = \frac{z_0}{4\pi^2} \cdot \frac{\rho'^2 \theta'^2}{(\rho'^2 - 1)}$$